On the Statistical Behaviour of the Aggregate Interference from WSDs into DTT systems

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Abstract

A methodology to calculate the maximum allowed power for White Space Devices (WSD) operating in the Digital Terrestrial Television (DTT) frequency bands is defined in the recent regulatory statement produced by the United Kingdom Office of Communication [1]. In deriving the methodology, the usual procedure of dividing the DTT service area into cells was considered. The maximum allowed transmit power for all WSDs in a given cell was determined under the single entry assumption that the victim DTT receiver is being interfered only by the WSDs in that cell. They have assumed that aggregate interference would not be a problem in the short term.

To investigate the limits within which the aggregation of interference from WSDs exceeds the regulatory threshold established by the methodology in [1], a first study [2], in which the geographical locations of the WSDs are modeled as a two-dimensional Poisson Point Process (PPP) was made. In the study, the statistical behaviour of the aggregate interference (probability distribution function - PDF) was obtained via Monte Carlo simulation.

This paper presents an alternative method to compute the aggregate interference PDF, that is based on the analytical expression of the joint probability density function of the distances of the k nearest neighbours in a two dimensional PPP. Results are compared to those obtained using Monte Carlo simulation.

Keywords: White Space Device, Aggregate Interference, Stochastic Geometry, Cognitive Radio.

1. Introduction

Since last decade, the cognitive radio has been proposed to be a secondary system in digital TV frequency band, in order to share free channels (white spaces) and increase capacity to wireless communication services. Such radios are usually known as white space devices (WSD) and they should not cause harmful interference to digital TV primary services (DTT).

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In early 2015, the regulatory body of United Kingdom Office of Communication (Ofcom) published the first European regulation allowing for the operation of WSDs in the DTT allocated bands with the support of a geolocation database [1]. The criterion used for calculating the power limits is based on a maximum allowable degradation, in terms of a defined quality coverage parameter, that would guarantee that the DTT receivers will not be subjected to harmful interference. The quality coverage parameter used was the so called *location probability*. Some aspects of the methodology in [1] for calculating the WSD maximum power emission limits are presented in Section 2.

The Ofcom regulation [1] assumes that only one WSD radiates per pixel and per channel. It recognizes, however, that a WSD database may provide services to multiple WSDs in the same geographic area and the same DTT channels, resulting in an aggregation of interference. Nevertheless, Ofcom statement "believes that such aggregation of interference is unlikely to be problematic in the short term". The paper [2] assessed the limits within which the WSD aggregate interference power exceeds the regulatory threshold specified in [1] via Monte Carlo simulation of a two-dimensional Poisson Point Process (PPP).

In this paper, the statistical behaviour of the aggregate interference from WSD transmitters into a DTT receiver is calculated using the joint probability density function of the nearest neighbour distance in a two-dimensional PPP. In Section 2, the Ofcom method to determine the maximum *e.i.r.p.* (equivalent isotropically radiated power) allowed for the WSD transmissions, which is based on sigle-entry interference, is briefly described and the mathematical model used to determine the probability distribution function (PDF) of the aggregate interference is derived. An alternative method to compute the aggregate interference PDF, on the analytical expression of the joint probability density function of the nearest neighbours distances is presented in Section 3. Numerical results and conclusions are presented in sections 4 and 5, respectively.

2. Aggregate Interference

According the methodology applied in [2], the aggregate interference $i_{iagg,j}$ reaching Cell j, must satisfy the condition

$$P(i_{agg,j} > Z_j) \leq 0.01 \tag{1}$$

In each cell, the value Z_j is determined so that its *location probability* (a DTT coverage quality parameter) is not decreased by more than 7%

[1]. Once Z_j is known for all cells in the DTT service area, it is possible to determine, for each cell, the maximum value of *e.i.r.p.* that could be transmitted by all WSDs in the cell without producing, at the neighbouring cells, interference powers that do not satisfy (1). To do so, consider the geometry illustrated in Figure 1 and let E_{kj} be the *e.i.r.p.* transmitted by a WSD in Cell k in the direction of Cell j. The interfering power i_{kj} reaching a DTT receiver located at the center of Cell j is given by

$$i_{kj} = E_{kj} + g(\theta_j) - L_{kj} + \rho(\Delta f) \tag{2}$$

where L_{kj} is the propagation loss experienced by the WSD transmission from Cell k to Cell j (in dB), $g(\theta_j)$ is the DTT receiving antenna gain in the direction of Cell k (in dBi) and $\rho(\Delta f)$ represents the protection ratio (in dB) required to protect the wanted signal from interference when the interfering and the desired frequency channels are Δf Hz apart.

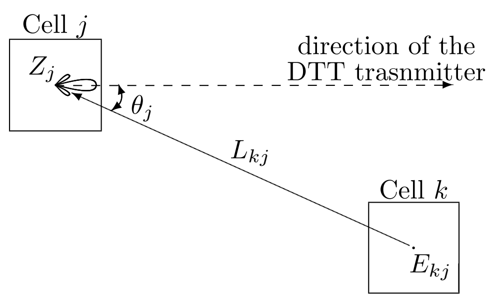

Figure 1 - Geometry for calculating E_{kj}

It is assumed that the propagation loss L_{kj} is given by the extended Hata model [3] which considers that, when expressed as a factor, the propagation loss has a lognormal distribution and, as a consequence, $L_{kj} \sim N(m_{kj}, \sigma_{kj})$ has a gaussian distribution. The linear relation in (2) indicates that i_{kj} is also a gaussian random variable. As a consequence,

$$P(i_{kj} > Z_j) = Q\left(\frac{Z_j - (E_{kj} + g(\theta_j) - m_{kj} + \rho(\Delta f))}{\sigma_{kj}}\right) \tag{3}$$

with $Q(\ )$ denoting the Q-function. Note that requiring the probability in (3) to be less than or equal to 0.01 is equivalent to require the argument of the Q-function to be greater than or equal to 2.33. This means that E_{kj} must satisfy the condition

$$E_{kj} \leq Z_j - g(\theta_j) + m_{kj} - \rho(\Delta f) - 2.33\,\sigma_{kj} \tag{4}$$

The maximum *e.i.r.p.* allowed for all WSDs in Cell k, considering all the surrounding cell j, is then given by

$$E_k = \min_j (Z_j - g(\theta_j) + m_{kj} - \rho(\Delta f) - 2.33\,\sigma_{kj}) \tag{5}$$

It is assumed that all WSDs in a given cell, say Cell k, operate with *e.i.r.p.* equal to E_k.

To calculate the aggregate interference power reaching the DTT victim receiver, consider the geometry in Figure 2 and let the inferfering power i_ℓ due to a single ℓ-th WSD, expressed in dBm, be written as

$$i_\ell = P_\ell - L_\ell + g(\theta_\ell) + \rho(\Delta f) \tag{6}$$

where P_ℓ denotes the *e.i.r.p.* transmitted by the interfering WSD (in dBm), L_ℓ is the propagation loss experienced by the WSD transmission (in dB), $g(\theta_\ell)$ is the DTT receiving antenna gain in the direction of the ℓ-th WSD (in dBi) and $\rho(\Delta f)$ is defined the same way as in (2). Again, the propagation loss L_ℓ is given by the extended Hata model [3], being then modeled as a gaussian random variable. This means that the interfering power i_ℓ is also a gaussian random variable.

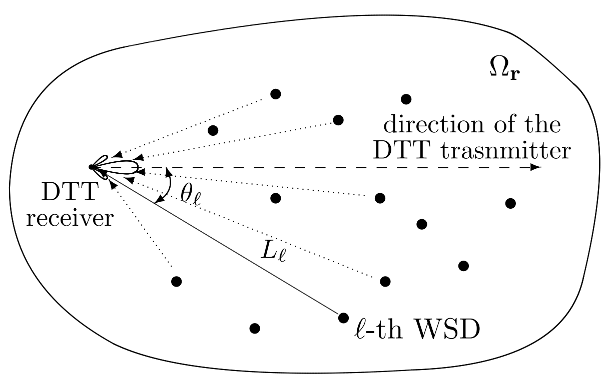

Figure 2 - Multiple interfering WSDs

Assuming N interfering WSDs, the aggregate interference power reaching the DTT receiver, expressed in dBm, is written as

$$i_{agg} = 10\log\left(\sum_{\ell=1}^{N} 10^{i_\ell/10}\right) \tag{7}$$

The expression between parentheses in (7) corresponds to a sum of statistically independent lognormal random variables. Determining the probability density function of the sum of lognormal random variables is a complex task. However, an approximation can be obtained by using the algorithm presented in [4] and numerically improved in [5]. In this algorithm the probability density function of the sum of lognormal random variables is approximated by a lognormal probability function. This means that, given the geographical locations $\mathbf{r} = \begin{pmatrix} r_1^T & r_2^T & \cdots & r_N^T \end{pmatrix}^T$ of all interfering WSDs (with $\{r_\ell, \ell=1,\ldots,N\}$ denoting the two-dimensional vector containing the longitude and latitude of the geographical location of the ℓ-th interfering WSD) the algorithm in [4] provides the mean and standard deviation of the aggregate interfering power, which has a normal (gaussian) distribution when expressed in dBm. Thus,

$$i_{agg} \sim N(m_{i_{agg}|\mathbf{r}=\mathbf{R}},\, \sigma_{i_{agg}|\mathbf{r}=\mathbf{R}}) \tag{8}$$

The unconditional Probability Distribution Function (PDF) of the aggregate interference power can be written as

$$F_{i_{agg}}(I) = P(i_{agg} \le I) = \int_{\Omega_\mathbf{r}} P(i_{agg} \le I | \mathbf{r} = \mathbf{R})\, p_\mathbf{r}(\mathbf{R})\, d\mathbf{R} \tag{9}$$

with $p_\mathbf{r}(\mathbf{R})$ denoting the probability density function of the WSDs geographical locations $\mathbf{r}$. Considering (8), the conditional probability in (9) can be written as

$$P(i_{agg} \le I | \mathbf{r} = \mathbf{R}) = 1 - Q\left(\frac{I - m_{i_{agg}|\mathbf{r}=\mathbf{R}}}{\sigma_{i_{agg}|\mathbf{r}=\mathbf{R}}}\right) \tag{10}$$

In [2], the integral in (9) was determined via Monte Carlo simulation with $p_\mathbf{r}(\mathbf{R})$ modeled by a two-dimensional PPP.

3. Aggregate Interference Using Nearest Neighbours

An alternative way to obtain the aggregate interference probability distribution function would be by ordering the terms of the summation in (7) in the decreasing order of their contributions to the aggregate interference and consider only the first K terms. The value of K would be determined so that the contribution of all other terms is negligible.

As a first step into this direction, this paper addresses the particular case in which the DTT receiver antenna has an omnidirectional pattern ($g(\theta_j) = G, \forall\, j$) and the maximum *e.i.r.p.* allowed for all the WSDs in a cell is the same for all cells ($E_k = E, \forall\, k$ with $E = \min_k(E_k)$). In this particular case, the interference contributions of the terms in (7) is directly related to the distances between the considered WSD and the DTT victim receiver and, as a consequence, the terms to be considered in (7) correspond to the K WSDs nearest neighbours to the DTT receiver.

Let then $\mathbf{r}_K = \begin{pmatrix} r_1^T & r_2^T & \cdots & r_K^T \end{pmatrix}^T$ denote the vector containing the geographical locations of the K nearest neighbours WSDs. The element r_k ($k\!=\!1,\ldots,K$) correspond a two-dimensional variable defining the geographical location of the k-th nearest neighbour WSD, which can be represented, in polar coordinates, by the distance d_k to the DTT receiver and the azimuth angle θ_k with respect to the DTT location. Thus, considering (9), we can write

$$P(i_{agg} \leq I) = \int\limits_{\Omega_{\mathbf{d}_K}} \int\limits_{\Omega_{\boldsymbol{\theta}_K}} P(i_{agg} \leq I | \mathbf{d}_K \!=\! \mathbf{D}, \boldsymbol{\theta}_K \!=\! \boldsymbol{\Theta})\, p_{\mathbf{d}_K \boldsymbol{\theta}_K}(\mathbf{D}, \boldsymbol{\Theta})\, d\mathbf{D}\, d\boldsymbol{\Theta} \quad (11)$$

where $\mathbf{d}_K = (d_1\, d_2 \cdots d_K)^T$ and $\boldsymbol{\theta}_K = (\theta_1\, \theta_2 \cdots \theta_K)^T$.

As in [2], the WSD locations are here modeled as a two-dimensional homogeneous Poisson Point Process (PPP). In this case, the distances d_k and the azimuths θ_k are statistically independent random variables [6]. As a consequence the random vectors $\mathbf{d}_K$ and $\boldsymbol{\theta}_K$ are statistically independent. Then

$$P(i_{agg} \leq I) = \int\limits_{\Omega_{\mathbf{d}_K}} \int\limits_{\Omega_{\boldsymbol{\theta}_K}} P(i_{agg} \leq I | \mathbf{d}_K \!=\! \mathbf{D}, \boldsymbol{\theta}_K \!=\! \boldsymbol{\Theta})\, p_{\mathbf{d}_K}(\mathbf{D})\, p_{\boldsymbol{\theta}_K}(\boldsymbol{\Theta})\, d\mathbf{D}\, d\boldsymbol{\Theta} \quad (12)$$

Again, given $\mathbf{d}_K = \mathbf{D}$ and $\boldsymbol{\theta}_K = \boldsymbol{\Theta}$, the aggregate interference probability density function can be calculated using the algorithm in [4] [5] and, as before,

$$i_{agg} \sim N(m_{i_{agg}|\mathbf{d}_K=\mathbf{D},\boldsymbol{\theta}_K=\boldsymbol{\Theta}},\, \sigma_{i_{agg}|\mathbf{d}_K=\mathbf{D},\boldsymbol{\theta}_K=\boldsymbol{\Theta}}), \quad (13)$$

implying that

$$P(i_{agg} \leq I | \mathbf{d}_K = \mathbf{D}, \boldsymbol{\theta}_K = \boldsymbol{\Theta}) = 1 - Q\left(\frac{I - m_{i_{agg}|\mathbf{d}_K=\mathbf{D},\boldsymbol{\theta}_K=\boldsymbol{\Theta}}}{\sigma_{i_{agg}|\mathbf{d}_K=\mathbf{D},\boldsymbol{\theta}_K=\boldsymbol{\Theta}}} \right) \quad (14)$$

The *independent scattering* (or *purely random*) property of an homogeneous Poisson Point Process [6] guarantees that the angles $\{\theta_k, k = 1, \ldots, K\}$ are statistically independent and uniformly distributed in the interval $[0, 2\pi]$. Then,

$$p_{\boldsymbol{\theta}_K}(\boldsymbol{\Theta}) = \prod_{k=1}^{K} p_{\theta_k}(\Theta_k) = \left(\frac{1}{2\pi}\right)^K \tag{15}$$

It is also known [7] that, for an homegeneous PPP, the joint probability density function $p_{\mathbf{d}_K}(\mathbf{D})$ of the distances of the K nearest neighbours is given by

$$p_{\mathbf{d}_K}(\mathbf{D}) = \begin{cases} \exp\left(-\lambda\pi D_K^2\right)\ (2\lambda\pi)^K\ \left(\prod_{k=1}^{K} D_k\right) & ; \quad \mathbf{D} \in \mathcal{S} \\ \\ 0 & ; \quad \mathbf{D} \notin \mathcal{S} \end{cases} \tag{16}$$

where λ is the point density of the PPP and

$$\mathbf{S} = \{\mathbf{D} \in \mathbb{R}^K : D_1 < D_2 < \cdots < D_K\} \tag{17}$$

Substituting (15) and (16) in equation (12)

$$P(i_{agg} \le I) = \lambda^K \int_0^\infty \int_{D_1}^\infty \cdots \int_{D_{K-1}}^\infty \int_0^{2\pi} \cdots \int_0^{2\pi} P(i_{agg} \le I | \mathbf{d}_K = \mathbf{D}, \boldsymbol{\theta}_K = \boldsymbol{\Theta})$$
$$d\Theta_1 \cdots d\Theta_K \quad \exp\left(-\lambda\pi D_K^2\right) \left(\prod_{k=1}^{K} D_k\right) \ dD_K \cdots dD_1 \tag{18}$$

Note that, under the conditions assumed in this section ($g(\theta_j) = G, \forall\, j$ and $E_k = E, \forall\, k$), the probability distribution function in (18) should approach the one in (9), as K increases. It is also important to observe that, while (9) is calculated via Monte Carlo simulation (in this case the reliability of the results depends on the number of simulation runs), (18) can be calculated using numerical integration (precision depends on discretization of the integration region).

4. Numerical Results

A comparison between the Nearest Neighbours method (NN) presented in Section 3 and the Monte Carlo simulation (MC) was done using a particular scenario in which three co-located DTT transmitters (operating in channels 24, 27 and 29 of the digital TV frequency band) provide services to users in a service area where a WSD system operates in the adjacent channel 26. The DTT receiving power in each cell, necessary to determine Z_j, was calculated using Recommendation ITU-R P.1546.

The victim DTT was assumed to be 35 km distant from the DTT transmitters. The maximum allowed *e.i.r.p.* for each cell in this sub-region was determined, using (5). The DTT victim receiver antenna follows an omnidirectional pattern with 3 dBi of gain and the adjacent channel protection ratio was determined according to "low" Class 1 type of WSD [8]. The extended Hata model was used to calculate the propagation loss assuming an urban clutter type. Antenna heights of 10 m and 30 m were considered for the victim DTT and interfering WSDs respectively.

The MC simulation, involving 1000 PPP samples, was used to calculate the WSDs aggregate interference PDF using (9). The WSDs aggregate interference PDF was also determined with the NN method using (18). Different values of K (number of nearest neighbours taken into account) were considered.

A first example evaluates the aggregate interference produced by multiple WSDs into DTT channel 27 with a density $\lambda = 0.1$ WSD/km^2. Results are presented in Figure 3. The curves in dash-dot red lines refer to NN method and the curve in blue solid line refers to MC simulation. Note that, as expected, the NN curves come closer to the MC curve as the value of K increases.

Another example involving a higher density of WSDs was also considered ($\lambda = 0.215$ WSD/km^2). The resulting curves are presented in Figure 4. Note that, in both examples (λ in the range 0.1 to 0.215 WSD/ km^2), the NN method with $K = 5$ (interference due to the first 5 nearest neighbours) provides results that can be considered a good approximation for the MC curve (differences < 2 dB). For larger values of λ, a larger number of nearest neighbours would be necessary in the NN method to approach the MC results. Also note that, as expected, the curves in Figure 4 are shifted approximately 3.3 dB ($10 \log(0.215/0.1)$) to the right, with respect to those in Figure 3.

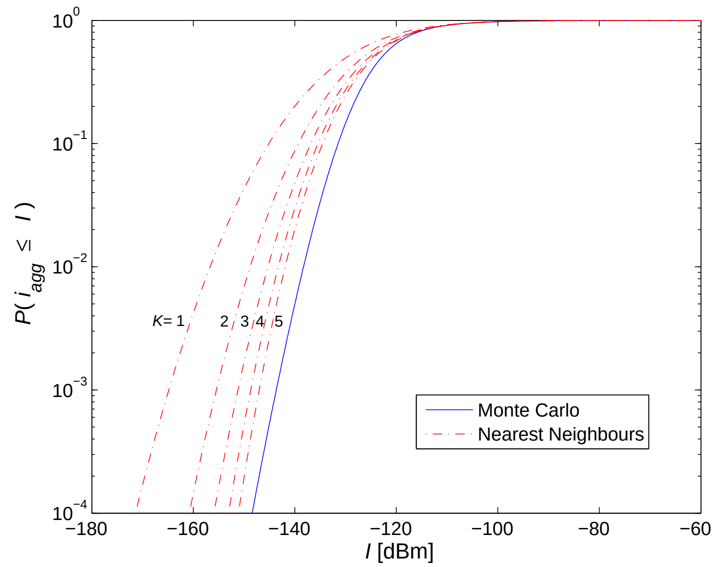

Figure 3 WSD aggregate interference PDF affecting channel 27 with $\lambda = 0.1$ using MC simulation and NN method.

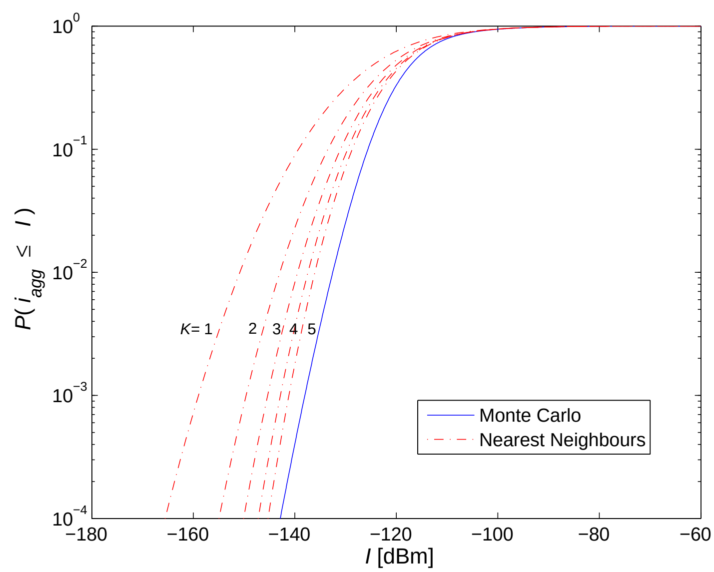

Figure 4 WSD aggregate interference PDF affecting channel 27 with $\lambda = 0.215$ using MC simulation and NN method.

5. Conclusion

This paper has developed a Nearest Neighbours mathematical model to assess the statistical behaviour of the interference produced by multiple WSDs into a DTT receiver. It calculates the aggregate interference PDF based on the joint probability density function (PdF) of the K nearest neighbour distances in a two dimensional homogeneous PPP. It takes advantage of the analytical expression available in the literature for this joint PdF to produce an integral closed-form expression for the aggregate interference PDF. Numerical examples allowed for a first comparison of the results produced by the proposed model with those obtained via a Monte Carlo simulation, demonstrating the feasibility of the new method. The next step would be to generalize the idea behind the proposed method to produce a model applicable to situations involving directional DTT receiver antennas.

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