



## A Global Optimization DFO-CRS Strategy for 1D Full Waveform Inversion

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### Abstract

We investigated the characteristics of the subsurface seismic exploration using Full Waveform Inversion (FWI) formulated as a nonlinear optimization problem in one dimension (1D). The FWI technique originally used mathematical methods based on the calculation of derivatives to optimize the objective function that quantifies the misfit between the data observed in seismic exploration and calculated by inversion. This entails a high computational cost and limited accuracy to local minima. In this work we adopt a Derivative-Free Optimization (DFO) methodology to find the desired global minimum. The technique used was the Controlled Random Search (CRS). We developed a FWI-CRS algorithm that numerically solves the 1D acoustic wave equation with the Finite Difference Method (FDM) and that minimizes the misfit by the CRS method, allowing to find the global minimum in the seismic inverse problem. The results using the FWI-CRS technique showed a good match with the actual model. The computational time was also reasonable. The objective function was shown to be very sensitive to small changes in the model parameters for the cases analyzed here.

**Keywords:** Full Waveform Inversion (FWI), Derivative Free Optimization (DFO), Controlled Random Search (CRS), Seismic Inverse Problem (SIP), Finite Difference Method (FDM).

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## 1. Introduction

The high complexity of underground geological structures makes the problem of seismic exploration a major challenge for the oil industry researchers. In fact, the occurrence of heterogeneities in all scales and limited knowledge (before drilling) about the properties of different parts of the underground are some of the causes for the difficulties found in the seismic exploration process. Nevertheless, it is possible to use the data obtained in seismic acquisition to solve the inverse seismic problem. This consists of taking recorded seismograms and use them to obtain a model that describes the properties of the subsurface to which the data is related [1].

In the search for more accurate subsurface features in seismic exploration, new inversion techniques were developed, as the Full Waveform Inversion (FWI). The FWI modeling is a numerical technique of inversion that determines the parameters that characterize an actual geological model [2]. In the resolution of a seismic inverse problem, the data set is formed by the wavefield, and we can use all information it contains, including phase and waveform [3].

The FWI traditionally uses derivative-based methods to optimize a cost function. This has a high computational cost and leads mostly to local minima [2]. In order to make it more robust and competitive, some authors have used Derivative-Free Optimization (DFO) methods [4]. In fact, the development and application of techniques based on DFO to optimize seismic inverse problems are of great interest [4, 5].

The methods based on DFO are generally successful and easy to implement. In this work we adopt the DFO methodology and a technique called Controlled Random Search (CRS) [5]. The specific objective of the paper is to present a FWI-CRS algorithm that, in the direct (forward) modeling, numerically solves the seismic acoustic wave equation in one dimension (1D) in time domain using the Finite Difference Method (FDM). The CRS technique was employed in the optimization process for solving the seismic inversion problem. This characterizes a FWI modeling with DFO optimization.

## 2. Mathematical FWI-CRS Formulation

In seismic inversion problems (SIP) as FWI, determining a subsurface model involves the use of an optimization method, instead of the direct inversion of a system of equations generated by a physical-mathematical modeling of the problem applied to recorded/observed data (seismogram) of reflected waves (seismogram), as someone would expect. The chosen optimization process repeats the forward modeling a significant amount of

times, trying to approximate an initial guess to a model that is compatible with the actual data. The forward modeling solves the wave equation, thus simulating the propagation of waves in an acoustic medium.

### 2.1 The Seismic Inverse Problem - SIP

The SIP can be understood as the application of a set of mathematical techniques to extract, from observed data, the correspondent physical parameters, considering a physical-mathematical model that explains the relationship between them [1].

Let  $d$  be a vector with the observed data,  $m$  a vector of parameters of the model (or the model itself), and  $G$  a mathematical operator that relates the model parameters  $m$  to the observed data  $d$ . These quantities are related by the equation:

$$d = G[m] \quad (1)$$

Thus, the SIP consists of, having  $d$  and  $G$ , finding  $m$ .

### 2.2 The Seismic Direct Problem - SDP

The SDP consists basically of determining the wavefield propagation (calculated data) from a set of parameters (model parameters) that characterize the geological medium, through the mathematical equation (wave equation) that relates them, Equation (1). With this, it is possible to predict whether the observed data, recorded on a seismogram, matches a given actual model. Those answers will be used in an inversion scheme that seeks to identify the model parameters that best approximate the observed data.

The FWI method is a mechanism that iteratively improves a particular initial model of the subsurface (initial model parameters). In each iteration, a search direction and a step is calculated and the model updated with it [3].

Analyzing the FWI as an optimization problem, the most common objective function used in it, aiming to quantify the misfit between the calculated amplitudes and the observed reflected P wave, is the least squares function. Therefore, the best fit of the calculated data to the observed one is associated with the minimum value of  $\phi$ , given by the expression [2]:

$$\phi = \frac{1}{2} \|A^{obs} - A^{calc}\|_2^2 \quad (2)$$

where,  $A^{obs}$  represents the amplitudes observed on a recorded seismogram; and  $A^{calc}$  contains the amplitudes of a synthetic calculated seismogram.

## 2.3 Uni-dimensional (1D) Modeling

### 2.3.1 The forward modeling applied to the seismic problem

The forward problem corresponding to the mathematical model of propagation of an acoustic wave, wave equation (WE) in one dimension, along with their initial values (IV) and boundary conditions (BC) are given by the mathematical operator  $\mathfrak{S}(m)$ , represented by the set of equations as follows:

$$\mathfrak{S}(m) = u \begin{cases} (WE) & \frac{1}{c^2(x)} \frac{\partial^2 u(x,t)}{\partial t^2} - \Delta u(x,t) = f(t) \delta(x - x_s) \\ (IV) & u(x,t) = 0, \text{ for } t \leq 0 \\ (BC - ABC) & \begin{cases} \frac{1}{c(x_0)} \frac{\partial u(x_0,t)}{\partial t} - \frac{\partial u(x_0,t)}{\partial x} = 0 \\ \frac{1}{c(x_L)} \frac{\partial u(x_L,t)}{\partial t} - \frac{\partial u(x_L,t)}{\partial x} = 0 \end{cases} \end{cases} \quad (3)$$

where  $c$  is the propagation velocity in the wavefield,  $u$  is amplitude or pressure.  $\delta(x - x_s)$  is the Dirac delta function.  $f(t)$  is the source function (Ricker wavelet). The variable  $t$  is the time window of the simulation (in seconds) [6].

For BC, we use the Absorbing Boundary Conditions (ABC). This condition is a mathematical artifice that we introduced in the modeling of WE to prevent unwanted reflections at frontiers of a computational domain [2].

### 2.3.2 Discretization by Finite Differences

The SDP is described by Equation (3), which are solved numerically. There are several methods that can be used for this purpose: Finite Element Method (FEM), Finite Difference Method (FDM), among others. In the present paper, we use the FDM. The essence of this numerical method is the discretization of the continuous domain into a mesh of nodes, size  $\Delta x$  with  $n_x$  points [6]. Such a discretization of the derivative is made using the Taylor series. After working the discretized equations, we get to an explicit scheme for FDM, which provides the values of  $U$  in the future time  $t + 1$ , in function of  $U$  at the present time  $t$  and at the previous time  $t - 1$ .

$$U_1^{t+1} = (1 - \lambda_1) U_1^t + \lambda_1 U_2^t, \text{ with } \lambda_i = c_i \frac{\Delta t}{\Delta x}, \text{ for } i = 1, \dots, n_x \quad (4)$$

$$U_i^{t+1} = \lambda_i^2 U_{i-1}^t + (2 - 2\lambda_i^2) U_i^t + \lambda_i^2 U_{i+1}^t - U_i^{t-1} + \sigma_i^2 f_i^t, \text{ } i = 2, \dots, n_x - 1 \quad (5)$$

$$U_{n_x}^{t+1} = \lambda_{n_x} U_{n_x-1}^t + (1 - \lambda_{n_x}) U_{n_x}^t, \text{ with } \sigma_i = c_i \Delta t, \text{ for } i = 2, \dots, n_x - 1 \quad (6)$$

These equations represent an evolution scheme of the temporal variable where, for each time step, the solution  $U(x, t)$  is found in an explicit way.  $\Delta x$  and  $\Delta t$  are the steps in space and in time of the FDM discretization.

## 2.4 Nonlinear Optimization Problem - NLP

For subsurface parameters determination in seismic reflection problems, it is necessary to invert the reflection coefficients of the reflected P wave. This characterizes a nonlinear optimization problem that consists in finding the parameter vector  $m = [m_1, \dots, m_m]$  so that the objective function  $\phi(m)$  is minimized [2, 4]. Many approaches can be used to solve that optimization problem, such as gradient or derivative-based ones like: Steepest Descent, Conjugate Gradient and Quasi-Newton. Derivative-Free Optimization (DFO) methods such as Random Jump Technique (RJT), Controlled Random Search (CRS), Nelder Mead Simplex (NMS) method, and Monte Carlo methods are another way [4].

Inverse problems are naturally formulated as a nonlinear optimization problem. Precisely, the optimization stage of the Seismic Inverse Problem (SIP) can be treated as a Nonlinear Programming Problem (NLP). In canonical form, a NLP is written as:

$$\text{minimize } \phi = \frac{1}{2} \sum_j \sum_k \left( A_{j,k}^{obs} - A_{j,k}^{calc} \right)^2 \quad (7)$$

$$\text{subject to } \begin{cases} \mathfrak{S}(m) = u \\ l_i \leq m_i \leq p_i, \text{ where } i = 1, 2, \dots, n \end{cases} \quad (8)$$

where, the index  $j$  refers to the number of recorded traces; the index  $k$  refers to the number of samples within the time window. The mathematical operator  $\mathfrak{S}(m)$  determines the wavefield  $u$  in parameter function  $m$ , size  $n$ , of the considered model.  $l_i$  and  $p_i$  are the associate constraints with each parameter  $m_i$ .

### 2.4.1 Controlled Random Search - CRS Method

Direct search is a nonlinear DFO approach, where the search space is directly exploited by seeking the best direction to minimize the cost function of the problem. The CRS is a type of direct search modified Monte Carlo method. It was created by *Price* and published in 1977, when proposing a new random search algorithm. The method can be seen also as a population algorithm, where a population of initial points is generated randomly and contracted iteratively toward a point expected to be the global optimum. CRS is a global search method, easy-to-implement and suitable for solving nonlinear inverse with reasonable efficiency [4, 5].

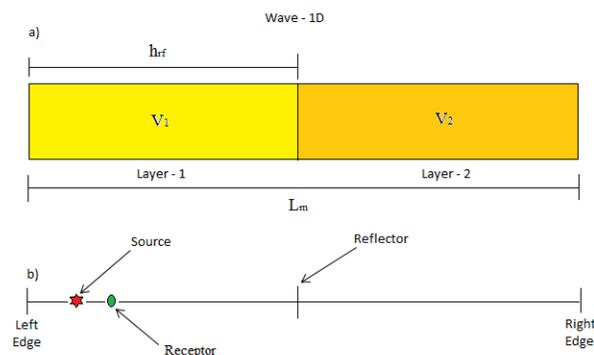
In the CRS algorithm, we consider that  $M$  is the number of model parameters (points) to be determined by the seismic inversion.  $\mathbf{B}$  is the parameters

space consistent with the constraint previously imposed, i.e., the restricted domain of the objective function, where all the possible solutions can be found.  $\mathbf{A}$  is a discrete subset of  $\mathbf{B}$  and is a reservoir of initial models, which will be updated during the iterative process of search for the optimum.  $L$  is the size of this reservoir. The value of  $L$  can be estimated by  $L = k(M + 1)$ , where  $k$  a suitable integer. The description of the algorithm is in [5].

### 3. Methodology

We use the acoustic wave equation in 1D and in time domain for modeling of seismic wavefield propagation. We also use the Derivative Free Optimization (DFO) technique as the optimization method for the inversion. Therefore, we developed a FWI-CRS algorithm that numerically solves the wave equation by FDM in SDP and that employs the Controlled Random Search (CRS) method to minimize the cost function of the SIP.

The geometry of the actual geological model consists of a line of unit length,  $L_m = 1.0$ . It is formed by two regions with velocity  $V_1$  and  $V_2$ . This model is called *layer-based velocity and reflector position model*,  $[VH]$  model. The synthetic seismic experiment had a seismic source, simulated by a Ricker wavelet with peak frequency equal to 10 Hz, located near the left edge of the model, at position 0.10. A receiver (geophone) was located at position 0.15. In this scheme, a reflector naturally appears in the center of the line. See Figure 1.



**Figure 1:** a) layer-based velocity model and reflector position  $[VH]$ ; b) edges scheme, reflector and sources/receivers.

The SDP was solved for the actual model with the intention of obtaining “observed” seismic data, recorded on a seismogram. In SIP, we do not know

the position of the layers, so it was added another parameter which is the position of the first layer. Thus, we have the set (vector) of parameters  $(V_1, V_2, h_{rf})$ , which values are iteratively obtained by seismic inversion. The values of these parameters are listed in Table 1.

The parameters  $(V_1, V_2, h_{rf})$  conform to certain restrictions imposed by the lower limits ( $L_{low}$ ) and upper ( $L_{up}$ ), established for each one of them. These limits are according to the constraints imposed by the problem model  $[VH]$  and are shown in Table 1.

Table 1: Parameters  $V_1$ ,  $V_2$  and reflector position  $h_{rf}$  with their lower ( $L_{low}$ ) and upper ( $L_{up}$ ) limits.

Limits/Parameters	$V_1$ (1.0)	$V_2$ (2.0)	$h_{rf}$ (0.5)
$L_{low}$	0.4	1.1	0.2
$L_{up}$	1.8	3.0	0.8

In the discretization process of the spatial domain with finite difference, we used a mesh with 201 nodes and a time window of 1.5 seconds.

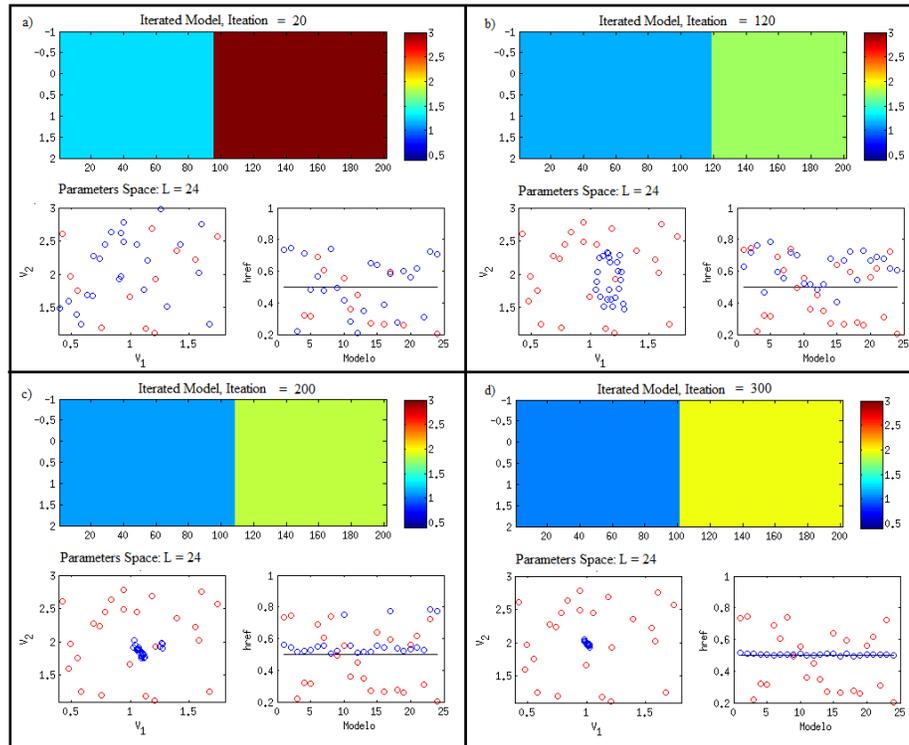
In order to evaluate the evolution of the optimization process towards the optimal solution, we adopted five values for  $It_{max}$ , the maximum number of iterations allowed,  $It_{max} = [200, 260, 300, 360, 460]$ . In addition, with  $M = 3$ , it was adopted  $L = 6(3 + 1) = 24$ . For the stop criterion of the CRS optimization, we used a tolerance value  $\epsilon = 10^{-4}$ .

#### 4. Results and Discussions

In this section are shown the results of the simulations for the SIP using DFO-CRS to optimize the misfit function between the observed data ( $d_{obs}$ ) and calculated ( $d_{cal}$ ). We analyze in details the case of  $It_{max} = 300$ .

The Figure 2 shows the results of the simulation corresponding to the SIP for some visits to the reservoir of solutions **A**. The visit consist to the random search a Simplex in the reservoir to be optimized by CRS. The parameters values  $V_1$ ,  $V_2$  and  $h_{rf}$  corresponds to the iterations number a) 20, b) 120, c) 200, and d) 300. The top of each quadrant displays the color scale corresponding to the layers velocity  $V_1$  and  $V_2$  and the reflector position  $h_{rf}$ . The bottom of each figure shows the models in its initial stages a), current b) and c) at last in final d). In this figure the red balls correspond to points (parameter vectors) in the initial reservoir. The blue balls identify the points in the current instant, during the iterations, including the final one. The update of reservoir parameters is such that the models with higher objective

function values, red balls, to be replaced by smaller value, blue balls, as the process of searching for the optimal solution evolves with the iterations.

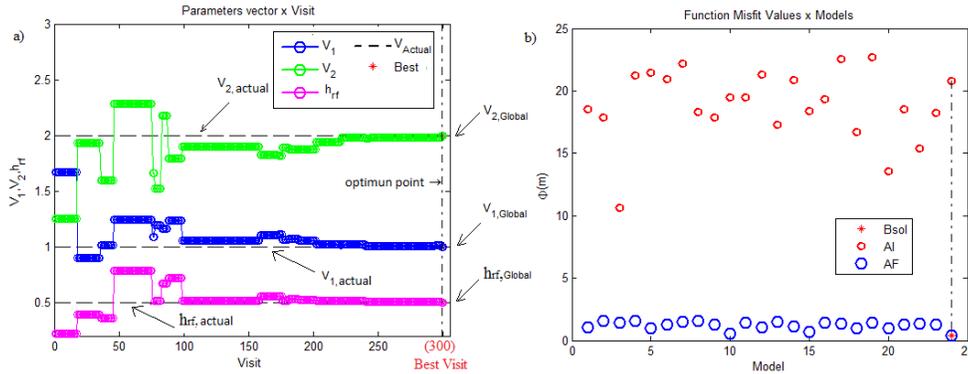


**Figure 2:** Found parameters values  $V_1$ ,  $V_2$  and  $h_{rf}$  at the iterations a) 20, b) 120, c) 200 and d) 300.

One can note that all models (blue balls) converge to the optimal point. The balls set comes from a distributed configuration to a considerably located one. At the final moment all models practically reach the same value. Finally, we say that the great Simplex, formed by all the elements of  $\mathbf{A}$ , reduced in size until it reached a sufficiently small value.

Figure 3a shows the model parameters evolve with visits. The model is identified by a vertical line drawn from each visit. For each visit, identify the optimal values achieved by the current method CRS for the parameters  $(V_1, V_2, h_{rf})$ , blue disc, green and magenta, corresponding to the model  $[VH]$ . The global optimum was achieved in visit 300 ( $it_{max}$ ). The Figure 3a shows the calculated values converge to the actual values. The Figure 3b shows the reservoir models in their initial and final stages. This figure shows that all models were replaced by updating the reservoir. And the last

visit from iteration 300 to the reservoir model of number 24 was replaced, now at point.



**Figure 3:** a) Evolution of  $[VH]$  model parameters ( $V_1$ ,  $V_2$ ,  $h_{rf}$ ), respectively blue, green and magenta lines, for each visit to  $\mathbf{A}$  in comparison with their respective actual values (dashed black line); b) initial models tank and final, with optimal solution (Bsol).

Table 2 shows, for the  $[VH]$  model, the observed and calculated (optimum global) parameters values ( $V_1$ ,  $V_2$ ,  $h_{rf}$ ), as well as the objective function and the computational time on a Intel i5 CPU machine.

Table 2: Actual and calculated parameters  $V_1$ ,  $V_2$  and  $h_{rf}$  for optimum global, objective function value and its computational time.

Values/Variables	$V_1$	$V_2$	$h_{rf}$	Cost function	Time
Real	1	2	0.5	–	–
Optimum	0.9999	1.9943	0.5007	0.4054	312.80

For the cases of  $it_{max}$  equals to 200, 260, 360 and 460, the same analysis was performed. It is shown in Table 3 that the parameters' values do not vary significantly for the examined cases. However, the objective function value showed to be more sensitive. Table 3 also reveals that the Size Great Simplex Initial (SGSI) value converges to the Size Great Simplex Final (SGSF) value, which by itself decreases as the  $it_{max}$  value increases. We can see that, for  $it_{max} = 460$ , the SGSF practically reduced to a point, which is the optimal solution and also coincides with the exact solution.

Table 3: Model  $[VH]$  with its parameters  $V_1$ ,  $V_2$  and  $h_{rf}$ , the objective function value  $\Phi(d)$  and values  $SGSI$  and  $SGSF$ .

$it_{max}/$ Variables	$V_1$	$V_2$	$h_{rf}$	$\Phi(d)$	$SGSI$	$SGSF$
200	1.0096	1.9885	0.5082	0.7404	1.0775	0.0848
260	1.0029	1.9928	0.5042	0.4904	0.9870	0.0679
300	0.9999	1.9943	0.5007	0.4054	1.0302	0.0676
360	1.0006	1.9962	0.5009	0.1867	1.0187	0.0263
460	1.0000	2.0000	0.5019	0.0034	1.0931	0.0003

## 5. Conclusions

In this research, a Controlled Random Search (CRS) algorithm for FWI was developed. We have solved the SDP by FDM and used the CRS method for minimizing the cost function, thus solving the SIP. The results found in this research using the FWI-CRS showed a good fit between the calculated model and the actual geologic model tested. Computational time was within the expected length. We also noticed that the parameters of the model are less sensitive than the ones of the objective function.

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