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Stability Analysis of the Equilibrium Point of the IS-LM Model for a Closed Economy

Leonardo Henrique Andrade do Nascimento[a], Silvio Antonio Bueno Salgado[b1], Danilo Machado Pires[c] and Felipe Andrade Velozo[d]

[a b c d] Institute of Applied Social Sciences, Federal University of Alfenas, Varginha, BRAZIL
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Abstract

In this paper, we analyze the stability of equilibrium, in the Lyapunov sense, of the dynamic IS-LM model, for a closed economy. Results of this analysis are supported by numerical simulations.

Keywords: IS-LM dynamic model, closed economy, equilibrium point, Lyapunov stability.

1. Introduction

The IS-LM model, characterized by a balance between the assets and services, and the monetary and financial markets, is, according to [7], an attempt to formalize the system mathematically in order to describe, in this context, the general theory of employment, interest and currency. According to [5], a determining factor for the discussion from the theoretical perspective is the importance of the interest rate in the generation of consumption and employment. In this sense, given the increase in interest rates, aggregate demand is reduced, due to its consumption and investment. The dynamic IS-LM model is defined by two first-order ordinary differential equations, which represent the trajectories of the goods and services market as well as the money market according to changes in the ordered pair (interest rate and product), respecting the premises emphasized by [5].

Many authors have studied the dynamics of the IS-LM model. Some important approaches are the following: In [3] an unbalanced version of the standard IS-LM model is constructed and used to scan the stability of the model. In [8] the authors development a model of exchange rate adjustments using the analytic and dynamic structure IS-LM-X. In [2] the authors scan the local dynamics of a general non-linear fixed price disequilibrium IS-LM model. In [1] the authors show the Kaldor-Philips business real cycle model with Hopf bifurcation theorem of the three dimensions, and the International trade model along with the dynamic IS-LM model by utilizing Hopf bifurcation theorem of the four dimensions.

The IS-LM model, in its dynamic version, evaluates the balance over time, making it possible to analyze the equilibrium stability of the model. An advantage of this approach is the feasibility of simulating a disruption in exogenous parameters (government spending and money supply), characterizing the application of an economic policy (fiscal and/or monetary), and then measuring the consequences of equilibrium.

In this work, we simulate the dynamic IS-LM model for a closed economy, for that a function was developed in Matlab software based on [6]. The function numerically solves the initial value problem associated with the system of linear ordinary differential equations that translate the dynamics of the model and returns

[1]E-mail Corresponding Author: silvio.salgado@unifal-mg.edu.br

several outputs that helps to evaluate the stability of the equilibrium. Furthermore, perturbations were made in the exogenous parameters which, supported by graphs, portray the dynamic to the model.

2. Preliminary

In this section, we present some basic concepts indispensable for the construction of this work. The concepts and results of ordinary differential equations were extracted from [4].

2.1 Ordinary Differential Equations

Consider the ordinary differential equation (ODE)

$$x^{'} = F(x) \tag{1}$$

where $x \in R^n$. One assumes that $F : R^n \longrightarrow R^n$ is a vector field of class C^k with $k \geq 2$. The solution of (1) starting at x at time $t = 0$ is denoted by $\gamma(t, x)$ or $\gamma(t)$. The map $t \mapsto \gamma(t, x)$ defines in R^n a curve passing through x at $t = 0$ that is called trajectory or orbit of (1) through x. A point $\overline{x} \in R^n$ is an equilibrium point of (1) if $F(\overline{x}) = 0$.

If $\gamma(t)$ it is a trajectory of (1) defined for all $t \geq 0$. Say $\gamma(t)$ is an stable if for all $\varepsilon > 0$ exists an $\delta > 0$ such that if $\psi(t)$ is solution of (1) and $\|\psi(0) - \gamma(0)\| < \delta$ then $\psi(t)$ is defined for all $t \geq 0$ and $\|\psi(t) - \gamma(t)\| < \varepsilon$, $t \geq 0$. On the other hand, if $\delta > 0$ such that $\|\psi(0) - \gamma(0)\| < \delta \Longrightarrow \lim\limits_{t\to\infty} \|\psi(t) - \gamma(t)\| = 0$, then γ is asymptotically stable.

An equilibrium point $\overline{x} \in R^n$ of the ODE $x^{'} = F(x)$ is stable if for all $\varepsilon > 0$, exists an $\delta > 0$ such that, if x_0 is an initial condition with $\|x_0 - \overline{x}\| < \delta \Longrightarrow \|\gamma(t, x_0) - \overline{x}\| < \varepsilon$ for all $t \geq 0$. An equilibrium point $\overline{x} \in R^n$ of the ODE $x^{'} = F(x)$ is atractive if exists an $\delta > 0$ such that $x_0 \in B(\overline{x}, \delta) \Longrightarrow \gamma(t, x_0) \longrightarrow \overline{x}$, when $t \longrightarrow \infty$. The set $B(\overline{x}, \delta)$ denoted an open ball centered on $\overline{x}$ and radius $\delta > 0$. An point $\overline{x} \in R^n$ is an asymptotically stable equilibrium point if $\overline{x}$ is stable and attractive.

Let

$$x^{'} = Ax \tag{2}$$

where $x \in R^2$, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and λ_1, λ_2 the eigenvalues of the matrix A. Denoted by $\sigma = a + d$ and $\Delta = ad - bc$ the trace and determinant of matrix A, respectively. The characteristic polynomial of A can be written in the form

$$\lambda^2 - \sigma\lambda + \Delta. \tag{3}$$

Theorem[4]: The equilibrium point of Equation (2) is asymptotically stable if, and only if, $\sigma < 0$ and $\Delta > 0$.

3. The Dynamic IS-LM Model

According to [5] the dynamic analysis of the IS-LM model is intended to provide a description of what occurs with the equilibrium points when there is a change in some exogenous parameter over time. This issue is important in that as the system may exhibit non-oscillatory or oscillatory evolution over time. The first assumption of the dynamic model, besides the validity of the static analysis, is an adjustment relation in the commodity market. If there is an excess demand $Z = C + I + G > Y$ the product increases; and if there is an oversupply $Z < Y$ the product decreases [5].

The consumption C is the sum of autonomous consumption a with the multiplication between the marginal propensity to consume b and the disposable income $Y_d = Y - T$. The level of taxes T is described by the sum of the fixed taxes T_0 with the multiplication between the sensitivity of the taxes in relation to the product θ and the total product Y. The level of investments I is the difference between the level of autonomous investments I_0 and the multiplication of the sensitivity of the investment in relation to the interest rate h with the interest rate r. The government spending is an economic policy variable, which will be considered exogenous to the analysis and will be represented by $\overline{G}$. This relation for the market of goods and services is given by

$$\frac{dY}{dt} = \alpha(Z - Y). \tag{4}$$

This assumption will also apply to the money market, so that if there is an excess demand for money $M_d > M_s$ the interest rate will increase. And if the market is oversupplied $M_d < M_s$ the interest rate will decrease. The velocity of adjustment in the market for goods and services is represented by β. Thus, we have the relation

$$\frac{dr}{dt} = \beta(M_d - M_s). \tag{5}$$

The second assumption of the dynamic model, also based [5], considers that at a point of disequilibrium the currency market adjusts faster than the goods market, that is, $\beta > \alpha$. In addition, [5] states that interest rates adjust rapidly as information is disseminated through the money market.

The demand of money M_d is constructed from the sum between the autonomous demand for money M_0 and the multiplication between the sensitivity of the demand for Money in relation to the product k and the total product Y. The demand for money still has a reducing share that is the multiplication between the sensitivity of the demand for Money in relation to the interest rate u and the interest rate r. The money supply is an economic policy variable and will be considered exogenous to the analysis and will be represented by M_s.

4. The Stability of Equilibrium in the IS-LM Model

In this section, we study the stability of the equilibrium point of dynamic IS-LM model. The dynamic IS-LM model is governed by the following system of differential equations

$$\begin{cases} \dfrac{dY}{dt} & = & \alpha(A_0 - YA_1 - hr) \\ \\ \dfrac{dr}{dt} & = & \beta(M_0 - \overline{M} + kY - \mu r) \end{cases}, \tag{6}$$

where $\overline{M}$ is money supply leavel and μ represents the sensitivity of the demand for money in relation to interest rates. The constant A_0 is represented by $a + bT_0 + I_0 + \overline{G}$ while the constante A_1 is represented by a constante $1 - b(1-\theta)$. The system given by (6) is in the form of a linear two-dimensional model $\mathbf{v}' = \mathbf{A}\mathbf{v} + \mathbf{B}$, where

$$\mathbf{v}' = \begin{pmatrix} Y \\ r \end{pmatrix}' \quad \mathbf{A} = \begin{pmatrix} -\alpha A_1 & -\alpha h \\ \beta k & -\beta\mu \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} \alpha A_0 \\ \beta(M_0 - \overline{M}) \end{pmatrix}.$$

For matrix A, we have $\Delta = \alpha\beta(A_1\mu + hk)$ and $\sigma = -\alpha A_1 - \beta\mu$. Moreover, the eigenvalues of matrix A are given by

$$\lambda_{1,2} = \frac{(-\alpha A_1 - \beta\mu) \pm \sqrt{D}}{2} \tag{7}$$

where $D = \alpha^2 A_1^2 - 2\alpha\beta(A_1\mu + 2hk) + \beta^2\mu^2$.

It is denoted by $(Y^*, r^*) = \left(\frac{A_0 - hr^*}{A_1}, \frac{M_0 - \overline{M} + kY^*}{\mu}\right)$ the equilibrium point of the system given by (6).

According to Theorem , (Y^*, r^*) is asymptotically stable if $\sigma < 0$ e $\Delta > 0$, that is

$$\begin{cases} -\alpha A_1 - \beta\mu & < & 0 \\ \alpha\beta(A_1\mu + hk) & > & 0 \end{cases}. \tag{8}$$

With respect to the first inequality

$$-A_1 < \frac{\beta\mu}{\alpha} \tag{9}$$

the α and β parameters that measure the respective rates of adjustment in each market have their domain restricted to only values large than 0. In addition, the sensitivity of the demand for money in relation to the interest rate must also be greater than zero. Thus, the right side of the inequality given by (9) will always be positive. By evaluating the signal of the parameter $A_1 = 1 - b(1-\theta)$, it is observed that $0 \leq \theta \leq 1$, where by the term $(1-\theta)$ will also be set in the same domain range of θ. The b domain is also comprised within this range, and hence the term $b(1-\theta)$ will be comprised within the same range. Thus, we conclude that $A_1 > 0$, so the inequality given by (9) is checked for any value assumed by the parameters.

Regarding the second inequality, we have

$$A_1 > -\frac{hk}{\mu}. \tag{10}$$

The parameters $h > 0$ and $k > 0$ measure, respectively, the rate of change of investments relative to the interest rate and the marginal rate of demand for money in relation to the product. In this way, the right side of inequality will always assume a negative value. From the result found by the equation 9 we have that $A_1 > 0$ for every parameter value. Thus, we conclude that the inequality 10 is always checked for any value assumed by the parameters and, therefore, the equilibrium point will be asymptotically stable for whatever values the parameters assume.

Table 1: Values of the trace, determinant and discriminant associated to the system given by (6).

σ	Δ	D
$-2,900$	$0,024$	$0,273$

Table 2: Eigenvalues and eigenvectors of matrix A.

λ_1	λ_2	$\mathbf{u_1}$	$\mathbf{u_2}$
$-0,354$	$-0,068$	$(-0,515 \quad 0,856)^T$	$(-0,515 \quad 0,856)^T$

5. Results and Discussion

In this section, we present numerical simulations. For the implementation in Matlab, the values for the parameters and initial conditions contained in [5] were considered. The adopted parameters are characterized as: $a = 15; I_0 = 10; \overline{G} = 25; \overline{M} = 8; \alpha = 0,05; \beta = 0,8; \theta = 0,25; b = 0,75; h = 1,525; k = 0,25; \mu = 0,5$. The conditions for the solution of the system by means of are $Y_0 = 75$ and $r_0 = 8$.

According to the Table 1, it can be inferred from the positive signal found by the discriminant that the behavior of the solutions in the vicinity of the equilibrium point at the origin is non-oscillatory, and then these solutions approach a point stable equilibrium. The combination of the results found on the determinant and trace, proves that the point of equilibrium is a stable node. This classification indicates that each initial condition has a trajectory in the phase plane, and for each time instant t has a point in the trajectory associated with it. In this case, the origin of the phase plane attracts all possible trajectories, since the point is asymptotically stable. Graphically, the origin represents a sink of the flow of trajectories in the phase plane.

The non-oscillatory behavior of the product and interest rate solutions that compose the model can be observed in the Figure 1 which considers as initial condition $Y_0 = 75$ and $r_0 = 8$. The initial instants of this economy show that there has been an instantaneous increase in the interest rate which has consequently caused a decrease in the total production of the economy, since an increase in the interest rate decreases the consumption levels and planned investments that are aggregated terms of national income. The behavior of the interest rate trajectory presents a higher speed of adjustment to the equilibrium. This result is explained by the assumed assumption that $\beta > \alpha$.

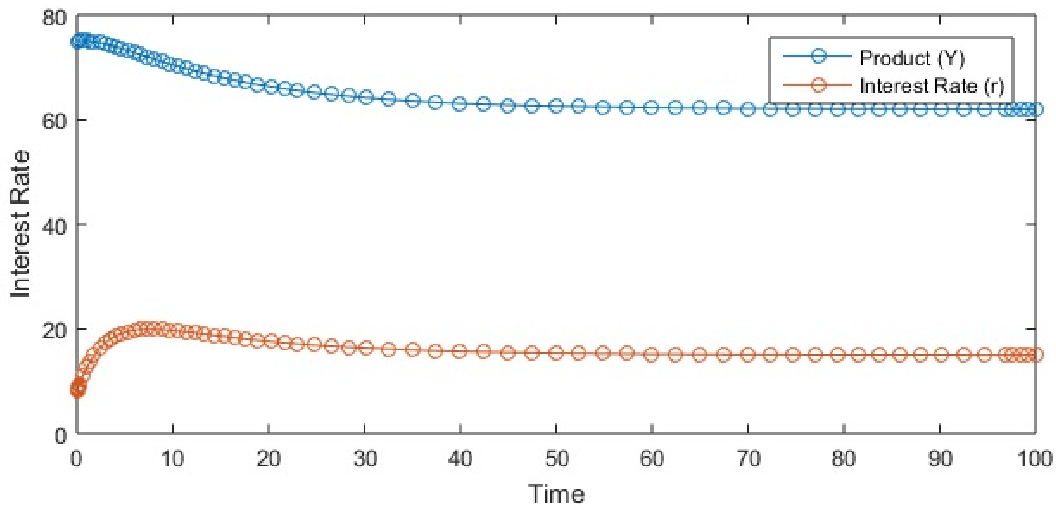

Figure 1: Evolution of the product and the interest rate.

The Figure 2 shows the evolution of the variables consumption, investment and product, starting from the initial conditions $Y_0 = 75$ and $r_0 = 8$, until the economy balance. Initially, there was a significant drop in investments. This decrease occurred due to the increase in the interest rate in the same period of time. This inverse relationship between investments and interest rates is explained by the expression that models the level of investments. The planned portion of investments depends on the interest rate. That is, given an increase in the interest rate the amount of planned investments increases, while the level of autonomous investments, which does not depend on the interest rate, remains the same. In this way, the difference that represents the level of investments reduces.

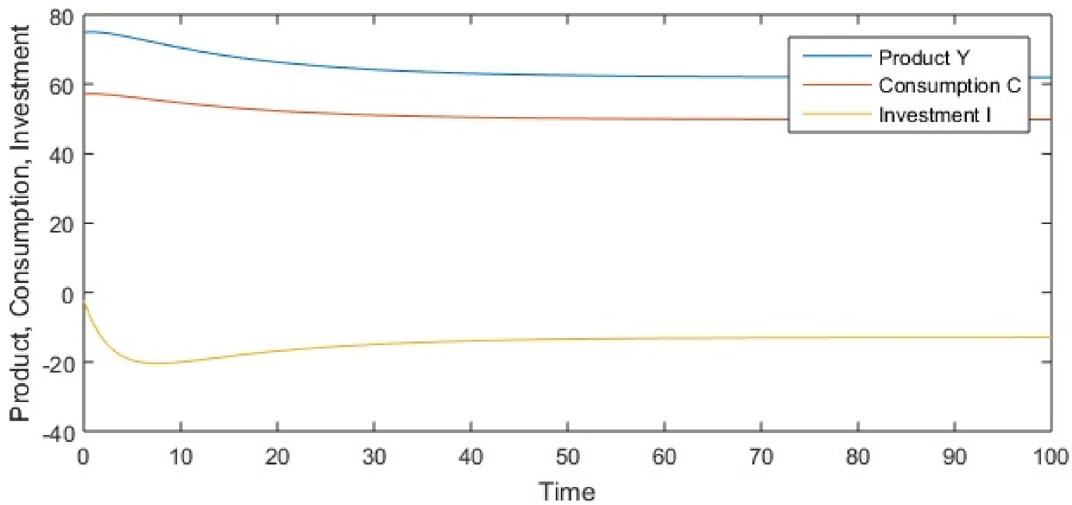

Figure 2: Evolution of product, consumption and investment.

Household consumption, on the other hand, does not have the same relation to the interest rate. This is because, mathematically, consumption is found by the sum of the autonomous consumption with the planned consumption. Autonomous consumption is a fixed quantity that depends neither on the total product nor on the interest rate. Already the planned consumptions are generated by the multiplication of the marginal propensity to consume and the disposable income, that depends only of the product. Graphically, we can observe that the consumption presented a slight fall. This decrease may have been generated indirectly by the increase in the interest rate. This is because, the increase in the interest rate, caused a decrease in the total level of production of the economy. This fall in the product, reaches the consumption, since the available income is measured by the difference of total product and tax collection.

From the analysis of the evolution of the variables aggregated to the product: consumption and investment, the importance of the interest rate in the IS-LM model should be highlighted. As seen above, a high interest rate discourage the planned investments and consequently the total level of investments, which in turn reduces the increase of the productive capacity of the companies, since the great majority of the companies look for investments to move the productive actions of the same. Regarding consumption, even if the interest rate is not directly influencing the mathematical expression, the level of consumption depends on the level of production of the economy. However, an increase in the interest rate reduces the quantity produced in an economy and, consequently, decreases the total consumption.

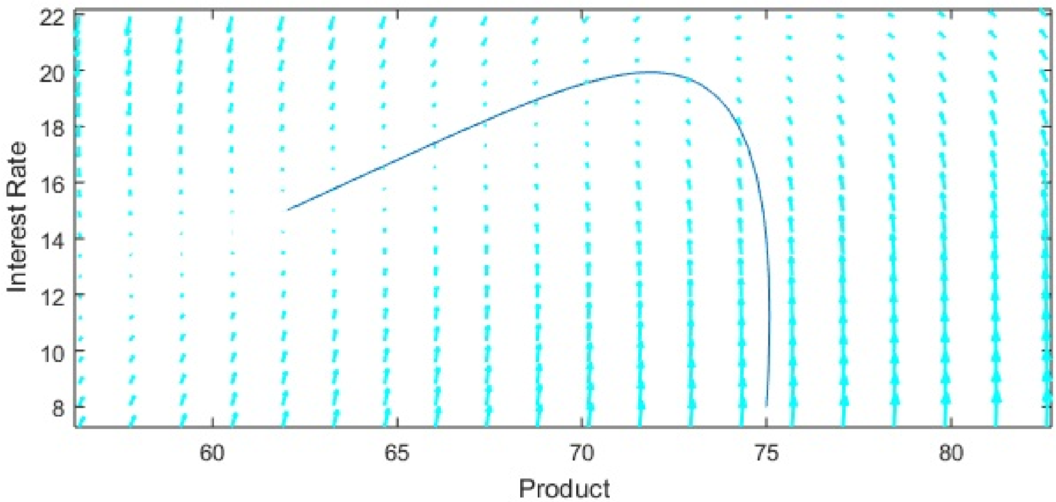

Figure 3: Phase-plane of the system given by (6).

The phase plane presented in Figure 3 depicts the temporal evolution of the equilibrium of the dynamic IS-LM model. This evolution emphasizes the greater speed of adjustments in the money market, since the interest rate is the factor of greater determination for this one. In the initial moments, the interest rate increases on a large scale, while the total production level remains almost constant. However, considering the equilibrium condition between markets, the economy in question needs lower interest rate levels for the model to remain balanced. The total production level begins to decrease almost the same as the interest rate begins to decrease. However, this does not imply that the decrease in the interest rate caused a decrease in total production. This is because the interest rate in time t, can lead to lower production levels in the instant t, however, the interest rate in time t probably has a greater influence on the level of production in the instants of future time, that is, $t+1, t+2, \ldots, t+n$. It is also worth mentioning the higher velocity of adjustment presented by the money market in relation to the market for goods and services, which possibly explains the great importance of the interest rate, for the balance adjustment presented in the previous discussions.

From the Figure 4 one can analyze the behavior of the equilibrium and infer about the variations occurring in the dependent variables over time, through the implementation of a contractionary monetary policy. This policy usually occurs when the economy is exposed to a high inflation rate, in order to reduce the supply by currency and, consequently, the level of prices. The reduction in supply by currency causes an increase in the interest rate. As a result, planned investment levels are reduced in line with the increase in interest rates. In addition, household consumption also declines, generating lower total production. The government adopts this type of measure with the intention of containing the planned expenses. A high interest rate means a higher opportunity cost of having the cash in hand, since it could be deposited in some financial

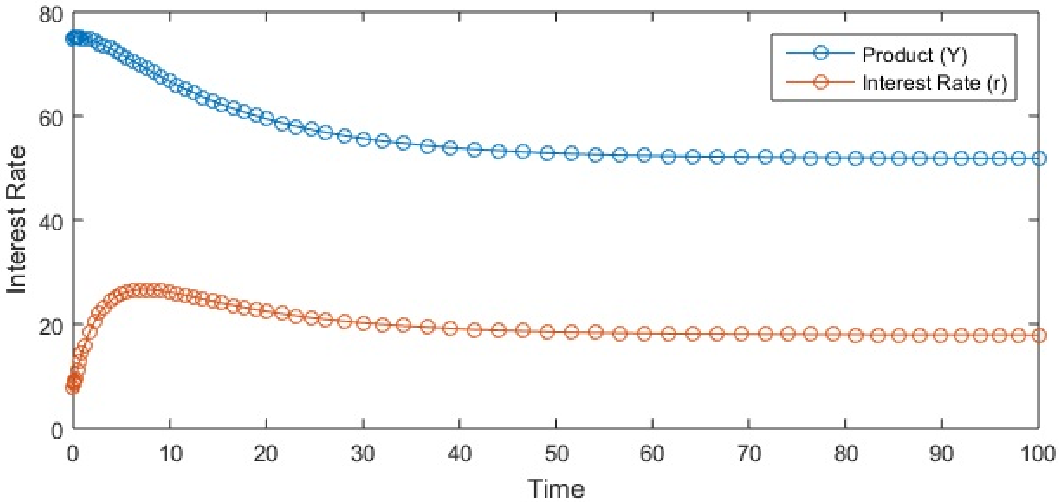

Figure 4: Contractionary monetary policy when the money supply varies from 8 to 4.

institution or invested in treasury bonds where interest income would stimulate the individual to invest it. For the monetary policy presented the new break-even point is $(51,53;17,92)$. Compared to the economy before the implementation of economic policy, the interest rate increases from 15 to $17,92$. In contrast, the product reduces from 62 to $51,83$.

6. Conclusion

We conclude that the equilibrium point of the dynamic IS-LM model is asymptotically stable for any variation in its parameters. For the simulation performed in the work, the eigenvalues found, associated to the coefficient matrix, were real since the discriminant signal was positive. In addition, the determinant signal was positive and the signal positive. Thus, we conclude that the break-even point is a stable node. The trajectories that describe the behavior of the interest rate and the total product tend to 0 as time t tends to infinity. That is, the origin of the plane attracts all possible trajectories that make up the system of equations.

It is worth mentioning that each initial condition produces a trajectory in the plane of the phases of the model, so, it is suggested for future work, an analysis including trajectories coming from different initial conditions. It is hoped that the trajectories will be in the same direction and will have the same oscillation for each initial condition. This is due to the asymptotically stable balance of the model.

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