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# Global wavelet analysis of impure chaotic dynamics

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**Abstract**

In this paper we introduce the *spectral complex index* in order to characterize the variability pattern of different chaotic dynamics. This index is obtained from the global wavelet analysis (GWA) of a chaotic signal with impurities (a stochastic noise component). The parameter is defined as the absolute distance between the powers with and without impurity (following the same principle of the KL divergence but not in the full entropic definition). The results suggest that, due to the Global Wavelet Spectrum logarithmic properties, this new spectral method provides a high sensitive measure for characterization of at least three different properties of the underlying dynamical process such as (i) the intrinsic noise level, (ii) the nature of the nonlinear deterministic rule (discrete or continuous) and (iii) the presence or not of spatial correlation.

**Keywords**: *Wavelet analysis, stochastic noise, chaotic dynamical systems, computational mathematics.*

## 1. Introduction

In the last two decades, simulation of chaotic systems has become one of the principal methods to investigate unexpected complex nonlinear regimes specially those observed from temporal and spatiotemporal variability which can be obtained, respectively, from simple nonlinear maps (discrete models) and nonlinear differential equations (continuous models). In fact, the phenomenological study of dynamics with irregular and unpredictable behavior have benefited from the development of few time series analysis techniques that are based on the chaotic dynamical systems theory [e.g [1, 2] and references therein]. In fact, popular techniques for characterizing chaotic variability are based on the phase space dynamics whose interpretation of correlation dimension and entropies, obtained from phase space reconstruction from a time series, is sometimes controversial and susceptible to several computational tasks that are difficult [e.g. [3]. Moreover, the calculation of the embedding dimension does not always permit a robust identification of the system when there is substantial noise in the deterministic component [8].

Recently, power-spectral analysis has been performed on colored random noise chaotic time series addressing the discussion on spectral characterization of hybrid (deterministic and stochastic) variabilities in terms of the stationarity and persistence levels of a given nonlinear fluctuation pattern [8, 9]. The characterization of a power-law spectrum is physically significant since many experimental measurements from widely different systems have approximate power-law spectra. The theory described by [10] shows that the

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stationarity and persistence levels of a self-affine time series can be characterized by means of the obtained power-law index $\beta$ if the Fourier power spectral density has frequency-dependent power-law. Another useful way of estimating power-laws is by means of the global wavelet spectrum (GWS) [10, 11]. In many cases, the wavelet analyses are the most satisfactory measures of power-laws ($\beta w$, in this context), particularly for data sets that have many nonstationary regimes [12, 13, 14]. A self-affine time-series is non-stationary with strong persistence when $\beta w > 1$ and stationary when $\beta w < 1$. The persistence is weak in the interval $0 < \beta w < 1$, uncorrelated for $\beta w = 0$, and antipersistence for $\beta w < 0$. Moreover, the global wavelet analysis (GWA) does not have red same same problems that are found in Fourier power spectral analysis, such as windowing, detrending, zero padding, etc [11]. As shown by [15, 16], power-law dependence on scale, given by $\beta$w, is a powerful estimation of the intrinsic properties of a complex time series generated by means of a system that can have intrinsic spatial coupling dependence and stochastic noise.

In this paper, we present a new method, based on the logarithmic properties of GWA [12, 13] combined with the concept of Kullback-Leiber divergence (KLD) [17], for analyzing impure chaotic time series generated from different chaotic dynamical systems. We apply this approach to characterize different discrete and continuous chaotic dynamics by means of the *spectral complex index* $\delta_S$, defined as the absolute distance between point-to-point powers redant the GWS of chaotic signal with and without impurity. To introduce and discuss our method we have generated canonical chaotic variability, with and without noise, from temporal domain: logistic and Henon maps and also from 1D spatial-temporal domain: Kuramoto-Sivashinsky [18] differential equation and Logistic Coupled Map Lattice [4, 5].

## 2. Data and Methodology

The data used in this work are representative on four distinct temporal variabilities of chaotic regimes, such as: (1) a logistic time series (1D chaotic variability regime) [2]; (2) a Henon time series (2D chaotic variability regime)[2]; (3) a time series obtained from Kuramoto-Sivashinsky equation (KSE)[4]; and (4) a time series obtained from a chaotic coupled map lattice (CML) [7]. The main properties of the signals analyzed in this work are: the (i) nonlinear chaotic regime, (ii) the nature of the temporal evolution (discrete or continuous) and (iii) the level of noise (here, we call the impurity level of signal). This third property (impurity) can be of two types: extrinsic and intrinsic. In this approach we have used only intrinsic noise. the chaotic signal is *pure* when the intrinsic noise is null[5].

Notice that, for the second property, the continuity implies space dependence, which is non-existent for the logistic and Henon signals as these are only defined in the temporal domain. Although, the first characteristic is common in all the systems, the nature of the temporal evolution is discrete only for the maps (logistic, Henon and logistic CML) while the KSE time series is generated from a nonlinear fourth order PDE. Figure 1 shows time series without noise for each chaotic signal (a) logistic, (b) Henon, (c) KS and (d) logistic 1D-CML while Figure 2 shows time series for the four signals with a given impurity level (around 10% for each case).

The one-dimensional KSE can be written as

$$\partial_t u = -\partial_x^2 u - v\partial_x^4 u - \partial_x u^2 \tag{1}$$

where $u(x,t)$ is subject to periodic boundary conditions $u(x,t) = u(x+2\pi,t)$ and $v$ is a 'viscosity' damping parameter. We adopt the spectral method by expanding the solutions in a discrete spatial Fourier series $u(x,t) = \sum_{k=-\infty}^{\infty} b_k(t)e^{ikx}$ that yields an infinite set of ordinary differential equations for the complex Fourier coefficients

$$\dot{b}_k(t) = (k^2 - vk^4)b_k(t) - ik \sum_{m=-N}^{N} b_m(t)b_{k-m}(t) \tag{2}$$

where $1 \leq k \leq N$, $N$ is the truncation order and the dot denotes the derivative. We choose $N = 16$ and introduce noise in the system by writing the equation for the Fourier mode $k = 1$ as

$$\dot{b}_1(t) = (1-v)b_1(t) - i\sum_{m=-N}^{N} b_m(t)b_{1-m}(t) + \delta(t) \tag{3}$$

where $\delta(t)$ is a stochastic term whose value is obtained at each time step by a random numbers generator with a gaussian distribution. We generate a noisy time series corresponding to the time variation of $u(x,t)$ at $x = 0.39$, with $v = 4.57$, which corresponds to approximately 1/3 of the variance of a noise free time series for the same regime at $v = 0.03$. Details on the algorithm and numerical solutions of this KS model are given by [4]. The model and the solutions for the CML analysed here are given by [5, 19].

Our wavelet analysis was conducted using a complex Morlet mother-wavelet. The power spectrum in function of time and scale (scalograms) for respective time series are plotted in the middle graph of Figures 1, 2 and 3. The time-average of the square modulus of the wavelet coefficients over the scalograms, is the so-called global wavelet spectrum (GWS) [11, 14]. The double log plot of their GWS in function of scale and the slope of straight line ($\beta w$) that fits the points appear on the right graph, and, as shown by [12, 15], from the estimation of the $\beta w$. In this paper, the global wavelet spectra complexity is given by a new distance parameter, between two statistical distributions, denominated *Spectral Complex Index* ($\delta_S$), given by

$$\delta(a) = \sum log[P_i/P_p], \tag{4}$$

where $P_p$ and $P_i$ represent power calculated through the GWS for pure and impure chaotic signal in function of scale $a$. this is a non-entropic form of KLD defined on the Global Wavelet Spectra. In this representation, due to the explicit presence of energy scales, we need to calculate only the energy differences between point-to-point powers of two probability density functions (PDFs). By taking into account the GWS for pure time series as the reference to calculate the $P_p$ values, we are estimating the ($\delta_S$) for impure time series for the chaotic systems introduced in this paper. The values are shown in Tables 1 and 2.

## 3. Results and Interpretation

One can realize that the first two temporal discrete chaotic signals are strictly anti-persistent with or without noise. This can be explained considering the nature of a nonlinear discrete process without real energy dissipation on a continuous spatial scale. Due to the continuous nature of the KS (whose dynamics has an intrinsic persistent (as a turbulent process the energy is distributed also on the spatial scales). See the values of $\beta w$ in Table 1.

The third discrete chaotic system (the Logistic 1D-CML) behaves as a hybrid persistent-anti-persistent power-law spectrum. We interpret this spectral behavior in the following phenomenology: due to the spatial coupling among the individual discrete maps, the energy is dissipated on the spatial scale imposed by the 1D-coupling reproducing a dissipative turbulent-like (persistent) for the more extensive scales (from the intermediate to the integral scale). On the other hand, this system has an anti-persistent spectral behavior for smaller scales, where the dissipation is uniform and homogeneous (practically, it does not depend on the scales) (see the curves on the right shown in Figures 1d and 2d).

The results Tables 1 and 2, show the efficiency of the *spectral complex index* for characterization of impure chaotic dynamics and its intrinsic complexity (presence or not of spatial correlation for energy distribution and the influence of intrinsic noise level). In particular, the results in Table 2 indicated that this new index is also sensitive to the persistence or anti-persistence (respectively, $\pm\beta w$) behavior of the signal. In the anti-persistent regime the impurity is directly proportional to the relative PDF distance. The inverse behavior is found for the persistent case.

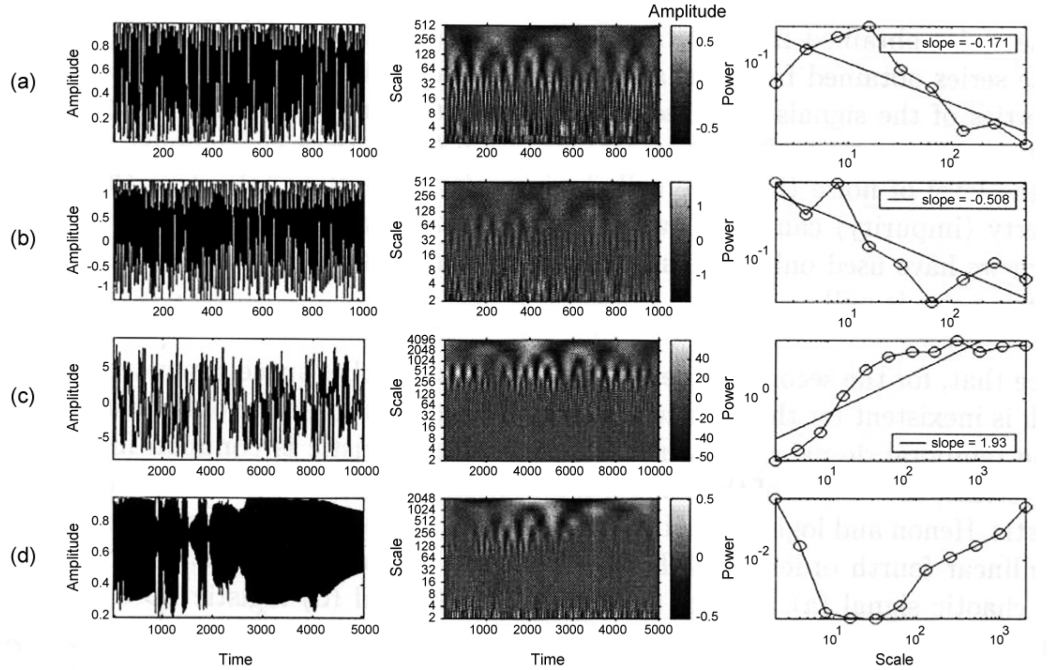


**Figure 1** - The time series without noise for each chaotic signal (a) logistic, (b) Henon, (c) K-S and (d) logistic 1D-CML. The center graph shows their scalograms and on the right the respective GWS.

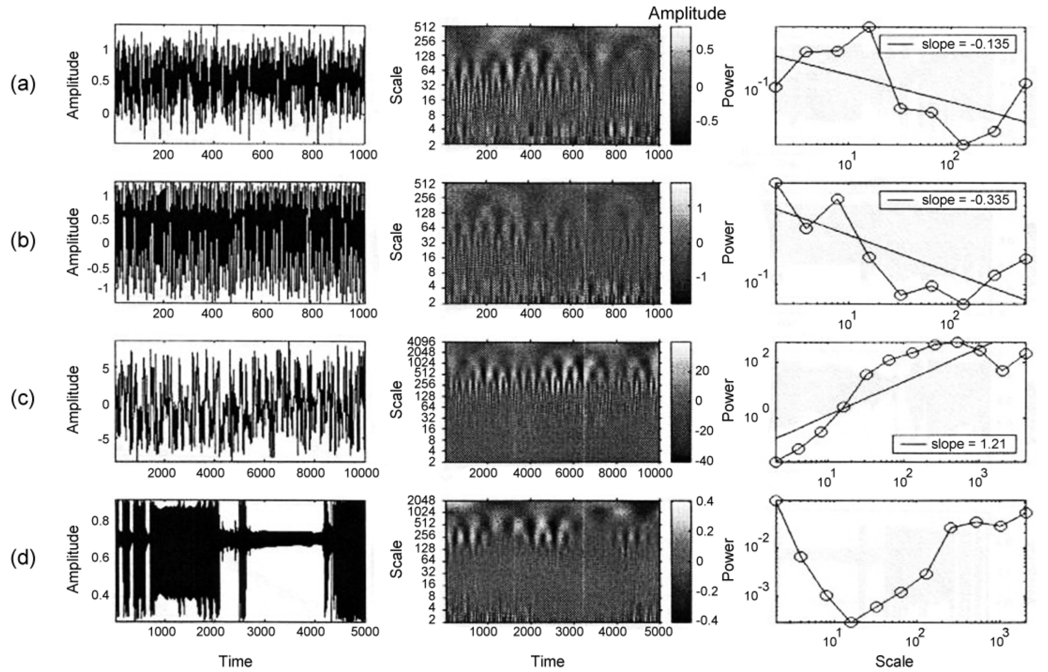


**Figure 2** - The time series with impurity level (10%) for each chaotic signal (a) logistic, (b) Henon, (c) K-S and (d) logistic 1D-CML. The center graph shows their scalograms and on the right their respective GWS.

In Figure 3 different time series for the Logistic 1D-CML are shown where each one has different impurity degree. Note that, we have shown only the persistent component of their GWS.

Table 1: General

| | $\beta w$ (pure) | $\beta w$ (impure) | $\delta_S$ |
|---|---|---|---|
| Logistic | -0.17 | -0.18 | 1.76 |
| henon | -0.51 | -0.34 | 1.42 |
| KS | 1.93 | 1.21 | 7.45 |
| CML | 0.24 | 0.31 | 2.73 |

Table 2: CML Anti-persistent (left) and persistent (right) Component

| 0-3 impurity level | $\beta w$ (pure) | $\delta_S$ | $\beta w$ (impure) | $\delta_S$ |
|---|---|---|---|---|
| CML0 | -2.93 | 0 | 1.15 | 0 |
| CML1 | -3.07 | -0.17 | 0.85 | -2.61 |
| CML2 | -2.98 | -0.64 | 0.95 | -2.55 |
| CML3 | -2.75 | -1.10 | 1.15 | -0.69 |

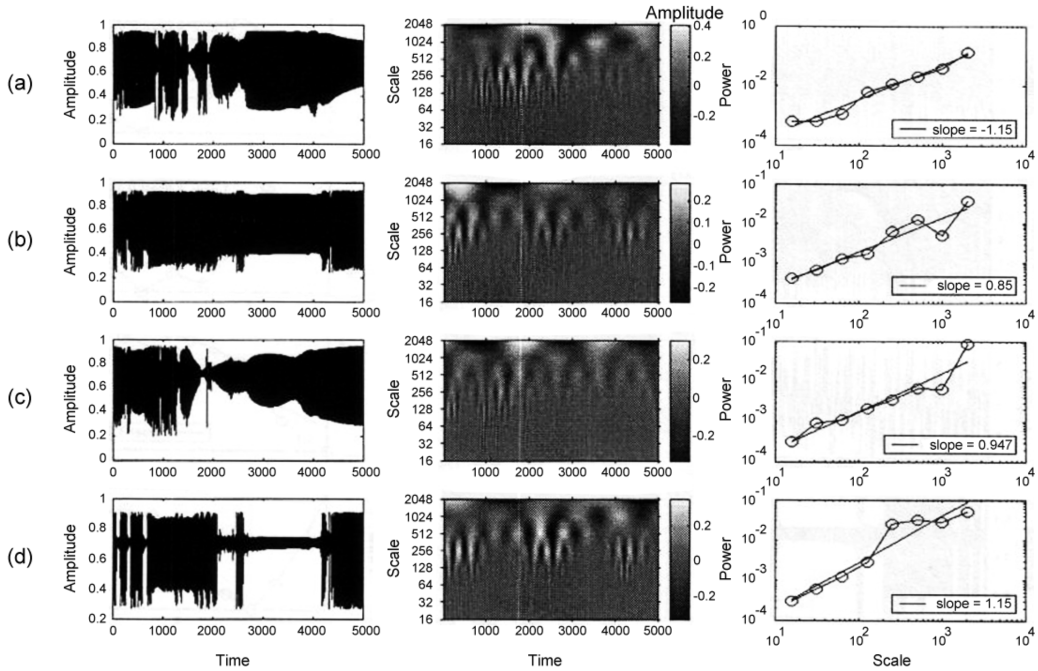


**Figure 3** - Four 1D-CML time series: (a) without noise and (b-d) with 3 different impurity levels. The center graph shows their scalograms and on the right their respective GWS.

## 4. Concluding Remarks

Characterizing complex dynamics properties from time series with the so-called noise impurity is a mathematical (and computational) task of great deal of attention from several applications of chaotic theory in nature. In this paper was presented a new technique based on GWS analysis provided consistent results, allowing the analysis of typical complex dynamics with noise. The methodology introduced here has been used to characterize the presence of intrinsic impurity, due to the noise in electronic components from real Chua's device. In this regard, this new analytic methodology can be also useful for chaotic modelling validation which will be reported further.

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## References

[1] Packard, N. H., J. P. Crutchfield, J. D. Farmer, R. S. Shaw, Phys. Rev. Lett. 45: 712-716, 1980.

[2] Kants, H., T. Schereiber. Nonlinear time series analysis, Cambridge University Press, UK, 1999.

[3] Ge, T., Cui, Y., Lin, W., Kurths, J., Liu, C., New J. of Physics 14: 2-11, 2012.

[4] Chian, A. C. -L., Rempel, E. L., Macau, E. E., Rosa, R. R., and Christiansen, F. Phys. Rev. E 65: 035203 (R), 2002.

[5] Beims, M. W., Rech, P. C., Gallas, J. A. C., Physica A 295: 276-279, 2001.

[6] Osborne, A. R., A. Provenzale. Physica D 35: 357-381, 1989.

[7] Kaneko, K. (1992), Overview of coupled map lattices, *Chaos: An Interdisciplinary Journal of Nonlinear Science,* 2(3), 279-282.

[8] Landa, P. S., M. G. Rosenblum, Physica D 48: 232-254, 1991.

[9] Savit, R., M. Green, Physica D 50: 95-116, 1991.

[10] Dorfman, J. R. An introduction to chaos in nonequilibrium statistical mechanics, Cambridge University Press, UK, 2001.

[11] Percival, D. and A. T. Walden. Wavelet Methods for Time Series Analysis, Cambridge University Press, UK, 2000.

[12] Amaral, L. A. N., A. L. Goldberger, P. C. Ivanov, and H. E. Stanley, Phys. Rev. Letters 81: 2388-2391, 1998.

[13] Malamud, B. D. and Turcotte, D. L., J. of Stat. Planing and Inference 80: 173-196, 1999.

[14] Flandrin, P., In: M. Farge, J. C. R. Hunt, J. C. Vassilicos (EDs.). Wavelets, Fractals, and Fourier Transforms, Oxford Univ. Press, 1993.

[15] Torrence, C. and G. P. Compo. A Practical Guide to Wavelet Analysis, Bulletin of the American Meteorological Society 79(1): 61-78, 1998.

[16] Neto C.R., Zanandrea A., Ramos F.M., Rosa R.R., Bolzan M.J.A., Sa L.D.A., Physica A 295: 215-218, 2001.

[17] Rosa R.R., Vats, H. O., Neto, C. Rodrigues, Fernandes, F. C. R., Ramos, F. M., Zanandrea, A., Rempel, E. L., Bolzan, M. J. A., Brito, R. C., Sawant, H. S. Advances in Space Researchs 29: 463-468, 2002.

[18] Wyner, A. D., Shannon Lecture, Typical sequences and all that: Entropy, Pattern Matching and Data Compression, 1994.

[19] Jousseph, C. A. C., S. E. de S. Pinto, L. C. Martins, M. W. Beims, Physica A 317: 401-410, 2003.