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Dynamical systems: an integrable kernel for resonance effects

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ABSTRACT

A suitable sequence of canonical transformations reduces the system of differential equations describing the orbital motion to an integrable dynamic system. Through this dynamic system, the motion of an artificial satellite subject to geopotential perturbations and resonances between the frequencies of the mean orbital motion and the Earth rotational motion is analyzed. The behavior of the motion of the satellite is analyzed in the neighborhood of the 2:1 resonances. The phase space of the resulting system is studied considering that one resonant angle is fixed. Simulations are presented showing the time-behavior of the semi-major axis of artificial satellites.

Keywords: resonance, artificial satellites, celestial mechanics, dynamics systems.

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1 INTRODUCTION

The influence of resonances on the orbital and translational motion of artificial satellites has been extensively discussed in the literature under several aspects. Among the resonances that have been studied we quote:

(i) resonance of the rotation motion of a planet with the translational motion of the satellite (Lima Jr. 1998; Formiga 2005);

(ii) sun-synchronous resonance (Hughes 1980);

(iii) spin-orbit resonance (Beletskii 1975; Hamill and Blitzer 1974; Vilhena de Moraes et al. 1990);

(iv) resonances between the frequencies of the rotational motion of the satellite (Hamill and Blitzer 1974);

(v) resonance including solar radiation pressure perturbation (Ferraz-Mello 1979).

In this work, the type of resonance considered is the commensurability between the frequencies of the satellite mean orbital motion and the Earth rotational motion. This case of resonance occurs frequently in real cases. In fact, in a survey from a sample of 1818 artificial satellites, chosen randomly from the NORAD 2-line elements (Celestrak 2004), about 85% of them are orbiting near some a resonances region.

The system of differential equations describing the orbital motion of an artificial satellite under the influence of perturbations due to the geopotential, can be described in a canonical form. In order to study the effects of resonances, a suitable sequence of canonical transformations can be performed reducing the system of differential equations to an integrable kernel (Lima Jr. 1998). This system is here integrated numerically. Simulations can show the behaviour of motion in the neighbourhood of the exact resonance. Some results considering the 2:1 resonances were already known but here, new results are presented for the 2:1. Graphics representing the phase space and the time variation of some keplerian elements are exhibited.

2 EQUATIONS OF MOTION AND THE CONSIDERED POTENTIAL

Using the classical set of Delaunay variables (L, G, H, l, g, h), the sideral time $\Theta = \omega_e t$, where ω_e is the angular speed of the Earth, and geopotential perturbations, the equations of motion can be written as (Osório 1973):

$$\begin{aligned} \frac{d(X, Y, Z, \Theta)}{dt} &= \frac{\partial F}{\partial(x, y, z, \theta)}, \\ \frac{d(x, y, z, \theta)}{dt} &= -\frac{\partial F}{\partial(X, Y, Z, \Theta)} \end{aligned} \tag{1}$$

were the Hamiltonian F is

$$F = \frac{\mu^2}{2L^2} + R_{\ell mpq} \tag{2}$$

and

$$\begin{aligned} R_{\ell mpq} = \sum_{\ell l=2}^{\infty} \sum_{m=0}^{\ell} \sum_{p=0}^{\ell} \sum_{q=-\infty}^{\infty} \frac{\mu^2}{L^2} \left(\frac{\mu a_e}{L^2}\right)^{\ell} \\ J_{\ell m} F_{\ell mp}(L, G, H) \times H_q^{-(\ell+1),(\ell-2p)}(L, G) \\ \cos \varphi_{\ell mpq}(\ell', g, h, \Theta) \end{aligned} \tag{3}$$

and the argument is given by:

$$\begin{aligned} \varphi_{\ell mpq}(\ell', g, h, \Theta) = q\ell' + (\ell - 2p)g \\ + m(h - \Theta - \lambda_{\ell m}) + (\ell' - m)\frac{\pi}{2} \end{aligned} \tag{4}$$

Also $\mu = GM \approx 3,986009 \times 10^5$ km^3/s^2 is the gaussian constant; $\lambda_{\ell m}$ is the longitude of the semi major axis of symmetry for the spherical harmonic (ℓ, m); $H_q^{(\ell+1),(\ell-2p)}(e)$ are the Hansen's functions and $F_{\ell mp}(i)$ are the Kaula's inclination functions.

3 METHODOLOGY

Let us represent by n the mean motion of the satellite and by $\alpha = q/m$ the commensurability from resonance condition

$$qn - m\omega_e = 0 \tag{5}$$

were q and m are integers.

The influence of the resonance on the orbital motion can be analyzed integrating a system of differential equations of the following type (Lima Jr. 1998; Formiga 2005):

$$\frac{dX_1}{dt} = -\sum_{p=s}^{\infty} B_{(2p+k)mp(\alpha m)}(X_1, C_1, C_2)\,\mathrm{sen}\,\varphi^*_{(2p+k)mp(\alpha m)}(x_1, \Theta_1) \tag{6}$$

$$\frac{d\varphi^*_{(2p+k)mp(\alpha m)}}{dt} = m\alpha\frac{\mu^2}{X_1^3} - m\omega_e - m\alpha\sum_{j=1}^{\infty}\frac{\partial B_{2j,0,j,0}(X_1, C_1, C_2)}{\partial X_1}$$

$$- m\alpha\sum_{p=s}^{\infty}\frac{\partial B_{(2p+k)mp(\alpha m)}(X_1, C_1, C_2)}{\partial X_1}\cos\varphi^*_{(2p+k)mp(\alpha m)}(x_1, \Theta_1) \tag{7}$$

Explicit functions relating $B_{(2p+k)mp(\alpha m)}(X_1, C_1, C_2)$ and $F_{lmpq}(X_1, C_1, C_2)$, with the keplerian elements can be found in Lima Jr. (1998).

4 NUMERICAL SIMULATIONS

Since Hansen's coefficients have been used this theory can be applied for orbits of any eccentricity below one. In this chapter we present simulations considering a few initial conditions arbitrarily chosen.

4.1 Results for resonance 2:1

Let us consider the case $e = 0,01$, $i = 4°$, $\varphi^* = 0$ (critical angle) and the harmonics J_2 and J_{22}. Numerical values for the Harmonics coefficients J_2 and J_{22} are given by JGM3.

The phase space for the dynamical system (6)–(7) is presented by Figure 1. This space is topologicaly equivalent to that of the simple pendulum. The behavior of the motion is here analysed in the neighborhood of the separatrix (a about 26562,48 km).

Figure 2 represents the temporary change of the semi-maxor axis in the neighbourhood of the resonance 2:1. For several values considered for the semi-major axes, distinct behaviors for the time-changes can be observed. The more the satellites approaches the region that we defined as a resonant region, the more the variations increases. It is remarkable the oscillation in the region between $a = 26560{,}0$ km and $a = 26562{,}48$ km, that characterizes paths for which the effect of the resonance is maximum for the case where $e = 0,01$ and $i = 4°$. For instance, for a small variation in the semi-axis, of about 10 m ($a = 26562{,}49$ km) an abrupt decrease in a is observed.

Figure 3 shows the temporary variation of the semi-major starting form initial conditions $a = a_o +$ 290 km and the $a = a_o +$ 480 km for $a_o = 26562{,}48$ km.

For the considered case, Figures 4 and 5 present, respectively, the behavior of the eccentricity ($0.0045 < \Delta e_{\max} < 0.0065$) and inclination ($0.000004 < \Delta i_{\max} < 0.000006$) in the neighborhood of the 2:1 resonance.

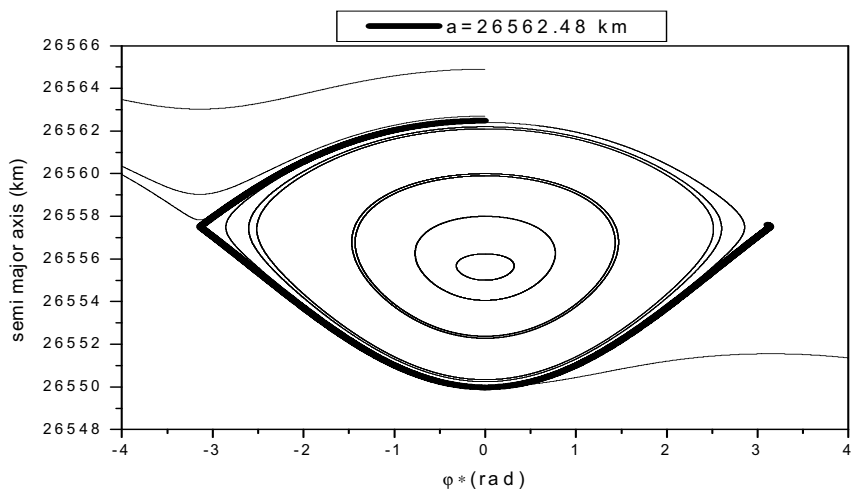

Figure 1 – Semi-axis *versus* critical angle: $e = 0,01$, $i = 4°$ and $\varphi^* = 0$.

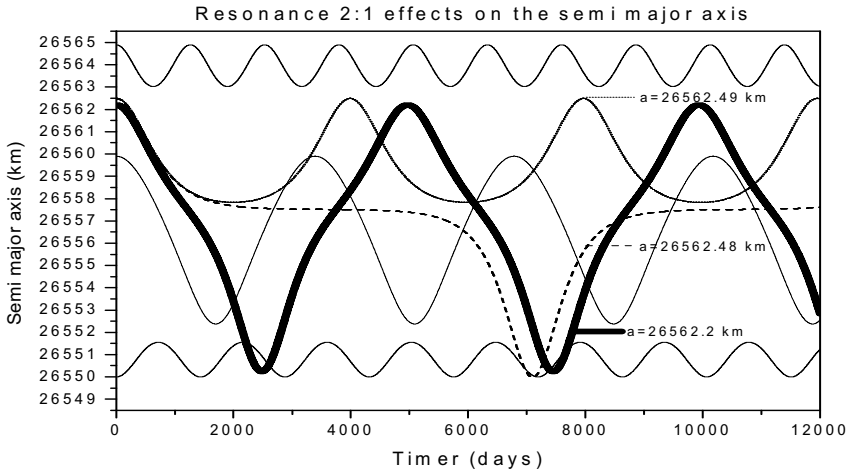

Figure 2 – Variation of orbits in the time: $e = 0,01, i = 4°$.

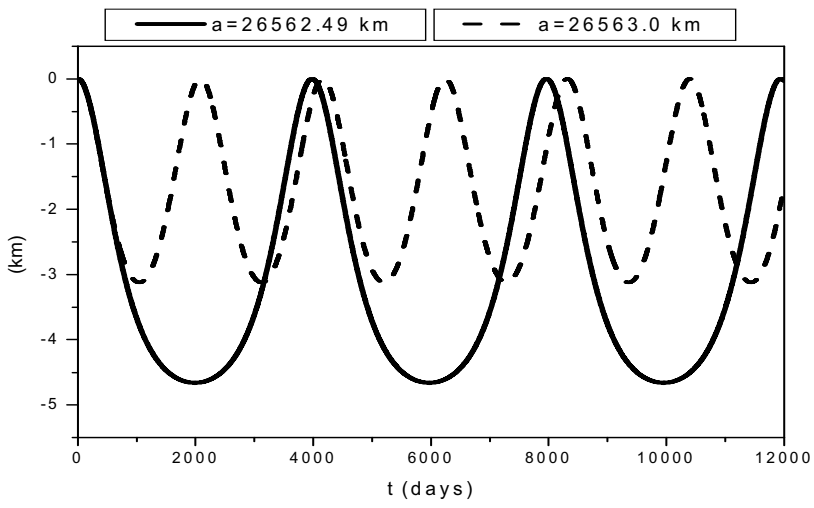

Figure 3 – Variation of orbits in the time: $e = 0,01, i = 4°, \varphi^* = 0°$.

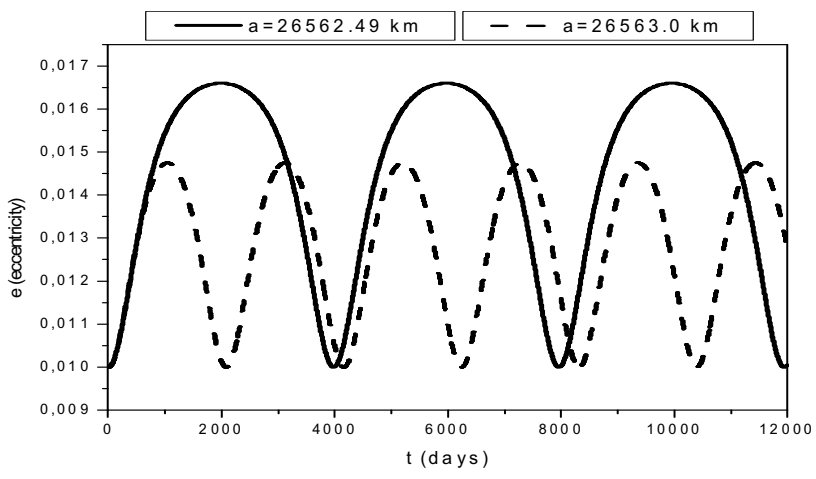

Figure 4 – Variation of eccentricity in the time: $e = 0,01, i = 4°, \varphi^* = 0°$.

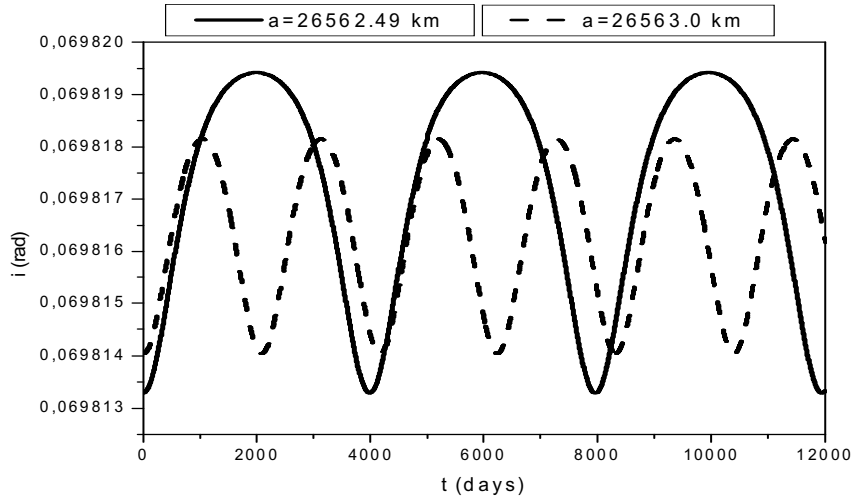

Figure 5 – Variation of inclination in the time: $e = 0,01$, $i = 4$, $\varphi^* = 0°$.

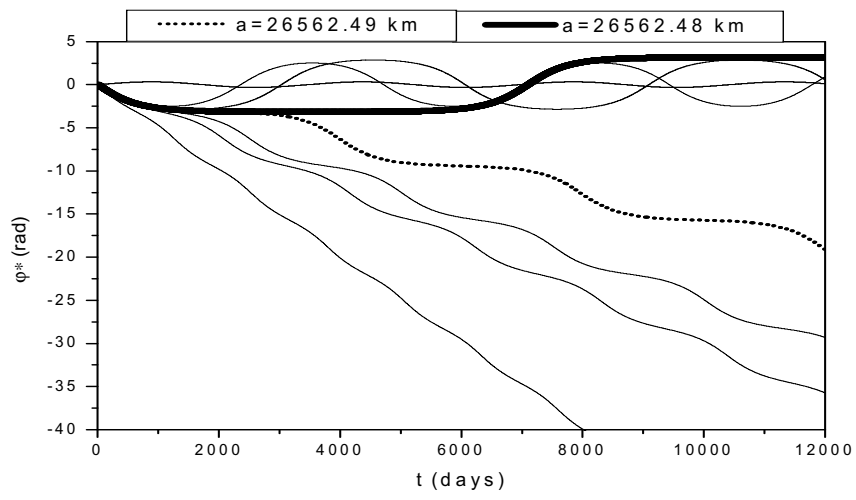

Figure 6 – Variation of critical angle in the time: $e = 0,01$, $i = 4$, $\varphi^* = 0°$.

The temporary variation of the critical angle is shown in Figure 6.

It is interesting to observe the behavior of the trajectories in the neighborhoods of the separatrix in the phase diagram given by Figure 6. Some paths were generated out of the resonant region. Still by, Figure 6, it can be observed the instant where the satellite is in the equilibrium position for $a = 26562{,}48$ km.

The stabilization of the orbit that appears after a period of 1420 days after a maximum oscillation (see Fig. 1), is related with a discontinuity produced by the resonance.

A possible region of stability for these conditions can also be observed by the behavior of the critical angle (Fig. 6). The satellite librates and circulate after suffering an abrupt discontinuity and starts to circulate in a region different from the initially considered.

5 CONCLUSIONS

In this work, based on Lima Júnior's theory (Lima Jr. 1998) a integrable kernel was found. The theory is valid for any type resonance p/q, and was here applied in a few cases.

The motion near the region of the exact resonance, is extremely sensitive to the small alterations considered. This can be indicative that these regions are chaotic.

This work provides a good approach for long period orbital evolution studies of satellites orbiting in regions where the influence of the resonance is more pronounced.

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REFERENCES

[1] BELETSKII VV. 1975. Resonance Phenomena at Rotations of Artificial and Natural Celestial Bodies. In: GIACAGLIA GEO (Ed). Satellites dynamics. Berlin. Verlang.

[2] CELESTRAK. 2004. "NORAD 2-line Elements". 10 Ago. 2004, <http://www.celestrak.com>.

[3] FERRAZ MELO S. 1979. Periodic orbits in a region of instability created by independent small divisors. In: NAGOZY E & FERRAZ MELO S (Eds). Natural and artificial satellite motion. Austin: University of Texas Press, p. 283–292.

[4] FORMIGA JKS. 2005. Estudo de ressonâncias no movimento orbital de satélites artificiais. 2005. 133f. Dissertação (Mestrado em Física) – Faculdade de Engenharia do Campus de Guaratinguetá, Universidade Estadual Paulista, Guaratinguetá, São Paulo, Brazil.

[5] HAMILL PJ & BLITZER L. 1974. Spin-orbit coupling: a unified theory of orbital and rotational resonance. Celestial mechanics, 9: 127–146.

[6] HUGUES S. 1980. Earth satellite Orbits with Resonant Lunisolar Perturbations. Resonances dependent only inclination. Proceedings of the Royal society of London serie A. London, 372(1745): 243–264.

[7] LIMA Jr. PHCL. 1998. Sistemas ressonantes a altas excentricidades no movimento de satélites artificiais. Tese (Doutorado em 1998), Instituto tecnológico de aeronáutica, São José dos Campos, São Paulo, Brazil.

[8] OSÓRIO JP. 1973. Perturbações de órbitas de satélites no estudo do campo gravitacional terrestre. Porto: Imprensa Portuguesa.

[9] VILHENA DE MORAES R & SILVA PAF. 1990. Influence of the resonance in gravity-gradient stabilized satellite. Celestial mechanics and dynamical astronomy, 47: 225–243.