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Computing hierarchical cosmological coupling constants from an alternative Seyfert-hyperbolic approach

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Abstract

New approaches to cosmological models, based on geometric and topological principles, may allow alternative interpretations for the dark content of the Universe and the inflation paradigm. In this work we use a cusp hyperbolic space glued to singular points of Seifert Fibered homologic spheres in order to avoid the cosmological point-like singularities (only in the glued parts). We show that some results of this approach can reproduce accurately, at least in terms of the hierarchy, a few recent measurements in experimental physics and observational cosmology. From the proposed infinite space with finite volume, we calculate the equivalent values for fundamental coupling constants in our current time for an alternative singularity-free Universe.

Keywords: *topology; cosmology; Thurston's theory; singularity-free models; computational mathematics.*

1. Introduction

In the theory of fundamental physical interactions, four are usually recognized: strong, electromagnetic, weak, and gravitational. Among the most important properties related to these interactions are the coupling constants, which describe their forces (or intensities), namely the strong force coupling constant (α_S), the fine-structure constant ($\alpha = \alpha_{EM}$ hereafter), the weak force coupling constant (α_W), and the gravitational coupling constant (α_G). These are phenomenological parameters which are usually obtained directly from experiments. Currently, there are no theoretical considerations which provide any explanation at least for the hierarchy of these constants. However, there has always been the hope that these constants could be computed in a deeper theory considering contemporary advances on the quantum field paradigm [1], [2].

In the present work we will consider the hierarchy of these coupling constants and will include also a fifth one, the cosmological constant, usually represented by the greek capital letter Λ. This constant was first introduced *ad hoc* in cosmological models by Einstein [3], as part of an additional term to his General Relativity Equations, to guarantee a static status for the Universe. In this context, Λ represents the curvature of the empty space. After the discovery of the expansion of the Universe [4], Einstein abandoned this concept, but Λ came back to scene in the end of XX$^{\underline{th}}$ century, now with the discovery of the accelerated expansion of the Universe (e. g. [5], [6]). In this new framework, Λ may be associated to a new content of the Universe, the one responsible for the current acceleration of the expansion, popularly called dark or vacuum energy, whose characteristics are very poorly known yet. The importance of the *dark energy* (or the cosmological coupling constant) is clear: Estimations of the content of the Universe reveal a fraction of about 70% for

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it respect to the total amount of matter-energy. From now on we will call all of above constants "coupling constants".

In order to solve the question about the hierarchy of the coupling constants. Efremov [7] recently proposed the use of topology. In his work he presented a manifold which admits 3-dimensional sections of Euclidean space-time cobordisms which are suitable for describing topology changes. Here we improve the idea of Efremov [7], [8] and Efremov & Mitskievich [9], [10]. Based on the splicing operation we construct a family of graphs, which contain Seifert Fibred (SF) Spaces and Hyperbolic Spaces. Graphs play the role of spatial sections in our three-dimensional spatial universe [11]. Here we develop an algorithm for computing the discrete volume of these space-graphs and also present the coupling constants calculated by the same method as [8] and [12]. This leads us exactly to the same hierarchy of constants but in a more accurate way than previous cited works, in the sense that we obtain better estimations for coupling constants that have been measured by physical and cosmological experiments.

In order to reach our goal we make the decomposition of the SF-Manifold in prime spaces[2]. This process includes a collection of cuts of a 3-dimensional space M^3 in two parts along 2-dimensional spheres such that the space M^3 is separated into two parts and neither of these is a 3-dimensional disc. This operation is the inverse of the connected sum. The result of gluing two 3-dimensional discs to each component of the boundary is to obtain a simpler 3-manifold and also the evolution of the Universe every time we do the process. Kneser [13] proved that such process is terminated after a finite number of steps. We call *prime components* of M^3 to each of the parts that were obtained as a result of this decomposition process. These are solely defined except by homeomorphisms (According to [14]).

As described by Eisenbud and Newmann [15], the reverse operation to the splice, discovered by Johanson [16] and Jaco & Shalen [17], can be used for making the decomposition. Such decomposition along the torus ($T_i^2 \subset M^3$) gives a set of 3-manifolds of which some are SF-Manifolds and the others allow hyperbolic structures. Previous works ([7] and [10]), namely 3-dimensional spaces which contain only SF-Spaces as parts, may be considered as a special case.

Using Thurston's conjecture ([18], [19], [20]) and the works of Jaco [21], Shalen [17] and Johanson [16], it is possible to glue a suitable hyperbolic cusp space in SF-Spaces.

Section 2 of this work presents the strategy for calculating the discrete volume of the considered manifolds, while sections 3 and 4 describe the algorithms for their computation, as well as the results obtained. In section 5 we discuss our main results and their implications.

2. Discrete Volumes

According to [11], a discrete volume in our approach is the number of tetrahedra from either the homology sphere or the hyperbolic cusp space. For triangulating SF-Spaces as homology spheres the reader can revise [12] and [11]. Here we will only consider the formula for the discrete volume that we need to take into account. Therefore, the number of tetrahedra to triangulate a toroidal neighbourhood of a singular orbit is:

$$N_{(sing)} = 3a_1a_2a_3. \tag{1}$$

Where a_1, a_2, a_3 are coprime relatives which satisfy the euler condition showed in [11].

Now, for a regular orbit, we have:

$$N_{(reg)} = 6a_1a_2a_3. \tag{2}$$

Considering a whole SF-Homologic Sphere $\Sigma(a_1, a_2, a_3)$, which have 3 regular orbits and 3 singular orbits, then the number of tetrahedra $N(\Sigma)$ is:

$$N(\Sigma) = 3 \times 3a_1a_2a_3 + 3 \times 6a_1a_2a_3 = 27a_1a_2a_3 \tag{3}$$

Following [22] the triangulation of S^3 is according to the structure of $\Sigma(a_1, a_2, a_3)$ and the cyclic cover $a_3 - folds$, branched along the torus knot $K(a_1, a_2)$.

[2] Prime spaces are the prime components of the decomposition.

Inasmuch as the torus where triangulation is constructed is the boundary of two solid torus, we have a triangulation of S^3, thus, the total number of tetrahedra in the triangulation of S^3 is $2 \times 3 \times a_1 \times a_2$. Recall that there are three possible combinations of Seifert invariants a_1, a_2, a_3 (namely a_1a_2, a_2a_3 and a_1a_3) so the average number of tetrahedra in S^3 is given by

$$N\left(S^3\right) = 2\left(a_1a_2 + a_2a_3 + a_1a_3\right) \tag{4}$$

this equation is not dependent of the permutations a_1, a_2, a_3.

From [11] we triangulated our cusp hyperbolic space obtaining also the number of generated tetrahedra, given by $N(H)$, being:

$$N(H) = 6a_1a_2a_3 \tag{5}$$

In the next section we will describe the computation of these numbers.

3. Computation of SF-Spheres

Continuing with the theory given in [11], in this section we present the algorithm approach for calculating the discrete volumes of our constructed manifolds. We will show that the Euler numbers given by SF-Homologic Spheres can reproduce the hierarchy of the low-energy coupling constants of the fundamental physical interactions [12]. From [11], the original topological treatment has been refining in order to obtain more accurate numbers for the coupling constants. Therefore, we introduce the following parametric definition:

Considering the parameter $\tau = \ell - n$, such that $\tau \in [-4, 4]$, the resulting family of SF- and SF-Homologic Spheres is

$$\{\Sigma_n^{(\ell)} :\, = \Sigma^{(n-\tau)}(q_{2n-1}, p_{2n}, p_{2n+1}) \mid n \in [0, 4]\}. \tag{6}$$

Note that if $\ell = n$, then we have $\left\{\Sigma^{(\ell)}\left(q_{i-1}, p_i, p_{i+1}\right) \mid i \in \overline{0, 8,}\ l \in [0, 4]\right\}$.

Now we rewrite the Euler number for (6) as:

$$e\left(\Sigma^{(n-\tau)}\right) = \frac{1}{[q_{2n+1}^{(n-\tau)}]}, \tag{7}$$

where $q_{2n+1}^{(n-\tau)} = q_{2n-1}^{(n-\tau)} p_{2n}^{(n-\tau)} p_{2n+1}^{(n-\tau)}$.

Now that once we have defined the "derivatives" with its corresponding Euler number, we can set-up the respective co-prime numbers of the SF-Homologic Spheres:

$$\Sigma^{(n-\tau)}\left(q_{2n-1}, p_{2n}, p_{2n+1}\right) = \Sigma\left(q_{2n-1}^{(n-\tau)}, p_{2n}^{(n-\tau)}, p_{2n+1}^{(n-\tau)}\right). \tag{8}$$

Note: The index $(n - \tau)$ indicate that we have the Seifert invariants of $(n - \tau)$ -th derivative of the SF-Homologic Sphere $\Sigma\left(q_{2n-1}, p_{2n}, p_{2n+1}\right)$.

Now if we take the parameter $\tau = 0$ in the family (6) of SF-Homologic Spheres that produce the hierarchy of coupling constants, is the family:

$$\left\{\Sigma^{(n)}\left(q_{2n-1}, p_{2n}, p_{2n+1}\right) \mid n \in [0, 4]\right\}. \tag{9}$$

Using (9) we rewrite $a^{(\ell)} = a^{(\ell-1)}\left(a^{(\ell-1)} + 1\right)$ as:

$$q_{2n+1}^{(k)} = q_{2n+1}^{(k-1)}\left(q_{2n+1}^{(k-1)} + 1\right) \qquad k = 1, ..., n. \tag{10}$$

In the following sub-sections we present the algorithms for compute the primary sequence of SFH-Spheres and its derivatives, in the next section, we develop the algorithm for coupling constants.

3.1. How to Generate the SF-Homologic Spheres

Algorithm 1: SF-Homologic Spheres (n_{max})
comment: n_{max} primary sequence of SFH-Spheres
$p \leftarrow$ generate prime numbers in order
for $n \leftarrow 1$ to n_{max}
$$\text{do}\begin{cases} [n;1;1] \leftarrow \sum(n;1;1) * \sum(n;2;1) \\ \sum[n;2;1] \leftarrow \sum(n;3;1) \\ [n;3;1] \leftarrow p[n+1] \end{cases}$$
comment: The third index in namely ℓ is used to save the derivatives.
comment: In this case 1, represents $\ell = 0$.

3.2. Computing Derivatives of the SF-Homologic Spheres

Algorithm 2: Derivatives ($n_{max}; l_{max}; \sum$)
comment: l_{max} derivation maximun order
for $i \leftarrow 1$ to n_{max}
do for $l \leftarrow 1$ to $l_{max}+1$
$$\text{do}\begin{cases} product \leftarrow [n;1;l] * \sum(n;2;l) * \sum(n;3;l) \\ \sum[n;1;l+1] \leftarrow \sum[n;1;l] \\ [n;2;l+1] \leftarrow product/[n;1;l] \\ [n;3;l+1] \leftarrow product + 1 \end{cases}$$

4. An Algorithm for Coupling Constants

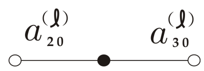
From the alternative manifold constructed in [11], each fundamental interaction is characterized by the following pair of parameters (n,ℓ), where $(n,\ell \in [0,4])$, and h which represents the level of a cusp hyperbolic space. These will be related to an assembly $E^{(\ell)}_{nh}$ of topological spaces composed by SF-Spheres and a cusp hyperbolic space each maintaining their own properties.

We begin by describing the assemblies $E^{(\ell)}_{nh}$ which represent the topology changes in the evolving universe and in turn also discrete volumes.

From the family:

$$\left\{\Sigma^{(\ell)}_n := \Sigma^{(n-\tau)}\left(q_{2n}-1, p_{2n}, p_{2n}+1\right) \mid n \in [0,4]\ , \tau \in [-4,4]\right\} \tag{11}$$

we take the first member $\Sigma^{(\ell)}_0\left(1, a^{(\ell)}_{20}, a^{(\ell)}_{30}\right)$ of the family which contains two regular and two singular orbits and we associate the following splice diagram (for a review of splice diagrams the reader can consult [23]):



Notation: the second subscript denotes the level h; for $n = 0$ we have $a^{(\ell)}_{20}$ and $a^{(\ell)}_{30}$. In the diagrams $\bullet$ is a regular orbit, $\circ$ is a singular orbit.

The triangulation of this SF-Sphere contains $N^{(\ell)}_0(\Sigma)$ tetrahedra and will be the discrete volume for this manifold. According to [11].

$$N^{(\ell)}_0(\Sigma) = 2 \times 3a^{(\ell)}_{10}a^{(\ell)}_{20}a^{(\ell)}_{30} + 2 \times 6a^{(\ell)}_{10}a^{(\ell)}_{20}a^{(\ell)}_{30} = 18a^{(\ell)}_{10}a^{(\ell)}_{20}a^{(\ell)}_{30}\ . \tag{12}$$

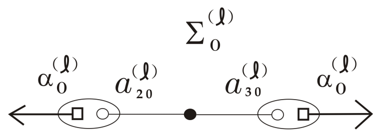
Now we will describe the develop of the assemblies. This is done by removing the torus neighbourhoods of singular orbits and introducing the torus part of a cusp hyperbolic space by a splice operation described in [15]. The number of tetrahedra for a hyperbolic manifold is:

$$N^{(\ell)}_n(H) = 6a^{(\ell)}_{1n}a^{(\ell)}_{2n}a^{(\ell)}_{3n}. \tag{13}$$

Then we have a new manifold denoted by $M_{0h}^{(\ell)}$

$$M_{0h}^{(\ell)} := H_0^{(\ell)} \underset{\alpha_1^{(\ell)} \qquad a_{20}^{(\ell)}}{\overline{\hspace{6em}}} \Sigma_0^{(\ell)} \underset{a_{30}^{(\ell)} \qquad \alpha_2^{(\ell)}}{\overline{\hspace{6em}}} H_0^{(\ell)} \tag{14}$$

and the corresponding splice diagram [here the arrows represent points to infinity and $\square$ represent a hyperbolic cusp space (H)]:



In this case we take the discrete volume of this manifold as the total number of tetrahedra generated by the partition. The volume is:

$$N_{0h}^{(\ell)} = 6a_{10}^{(\ell)}a_{20}^{(\ell)}a_{30}^{(\ell)} + 2 \times 6a_{10}^{(\ell)}a_{20}^{(\ell)}a_{30}^{(\ell)} + 6a_{10}^{(\ell)}a_{20}^{(\ell)}a_{30}^{(\ell)} = 24a_{10}^{(\ell)}a_{20}^{(\ell)}a_{30}^{(\ell)}. \tag{15}$$

With this process we remove singular orbits, which are exchanged by cusp hyperbolic spaces.

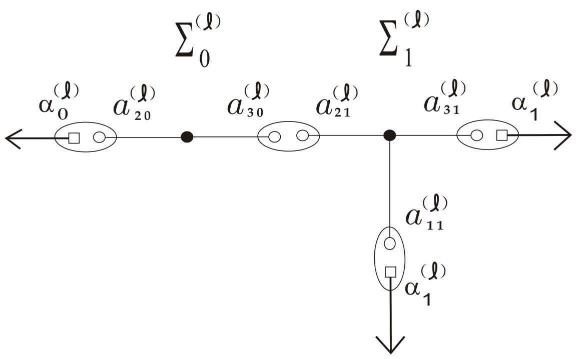
Now, consider the next SF-Sphere $\Sigma_0^{(\ell)}\left(1, a_{20}^{(\ell)}, a_{30}^{(\ell)}\right)$ of the family defined in (11) which contains two singular and two regular orbits (this is the zero level) and another SF-Sphere $\Sigma_1^{(\ell)}\left(a_{10}^{(\ell)}, a_{20}^{(\ell)}, a_{30}^{(\ell)}\right)$, with three singular and three regular orbits (this is level one).

In (14) we have glued two hyperbolic spaces, in order to glue inside $\Sigma_0^{(\ell)}$. The end of $a_{20}^{(\ell)}$ with $\Sigma_1^{(\ell)}$; but in $a_{30}^{(\ell)}$ we must first remove the hyperbolic space $h_0^{(\ell)}$ and then we will glue $a_{30}^{(\ell)}$ with $a_{21}^{(\ell)}$ applying a splice operation. After that, we add two hyperbolic spaces (also level one) in the singular orbits $a_{31}^{(\ell)}$ and $a_{11}^{(\ell)}$ of level one SF-Sphere.

For the construction of the assembly $\Sigma_{1h}^{(\ell)}$ we need to define the following manifold $M_{1h}^{(\ell)}$ as:

$$M_{1h}^{(\ell)} := \begin{array}{llll} H_0^{(\ell)} \underset{\alpha_1^{(\ell)} a_{20}^{(\ell)}}{\overline{\hspace{4em}}} & \Sigma_0^{(\ell)} \underset{a_{30}^{(\ell)} a_{21}^{(\ell)}}{\overline{\hspace{4em}}} & \Sigma_1^{(\ell)} \underset{a_{31}^{(\ell)} \alpha_3^{(\ell)}}{\overline{\hspace{4em}}} & H_1^{(\ell)} \\ & & \quad\big|{}^{a_{11}^{(\ell)}}_{\alpha_2^{(\ell)}} & \\ & & H_1^{(\ell)} & \end{array}. \tag{16}$$

In other words, we are indicating that the splice operation was made in a singular orbit S_3 of $\Sigma_0^{(\ell)}$ (which corresponds to $a_{30}^{(\ell)}$ invariant), and $\Sigma_1^{(\ell)}$ (corresponding to $a_{21}^{(\ell)}$ invariant) as well as $a_{31}^{(\ell)}$ and $a_{11}^{(\ell)}$ invariants. We remove one toroidal neighbourhood at each singular orbit and replace it by the corresponding $H_1^{(\ell)}$ space. For this we associate the splice diagram below:



Then the assembly will be defined by:

$$E_{1h}^{(\ell)} := \left\{ M_{1h}^{(\ell)}(R) \mid R \in \left[0,\ N_{0h}^{(\ell)} - 2N_0^{(\ell)}(H)\right] \right\}. \tag{17}$$

Here $N_{0h}^{(\ell)} - 2N_0^{(\ell)}(H)$ is the number of available tetrahedra for triangulate the regular tetrahedra. The $M_{1h}^{(\ell)}(R)$ components are obtained from $M_{0h}^{(\ell)}$ manifold using the connected sum R times among $M_{0h}^{(\ell)} \setminus TS_3$ sub manifolds. Where TS_3 is the tubular neighbourhood of a singular orbit S_3 extracted from SF-sphere $\Sigma_0^{(\ell)}$ using a splice operation. We have now:

$$N_1^{(\ell)}\left(S^3\right) = 2\left(a_{11}^{(\ell)}a_{21}^{(\ell)} + a_{21}^{(\ell)}a_{31}^{(\ell)} + a_{11}^{(\ell)}a_{31}^{(\ell)}\right). \tag{18}$$

Using the splice operation we removed singular orbits of $\Sigma_1^{(\ell)}$ and replaced them by a cusp hyperbolic space.

Now we need to glue $\Sigma_0^{(\ell)}$ and $\Sigma_1^{(\ell)}$, remove R tetrahedra from the sub manifold $M_0^{(\ell)} \setminus TS^3$ and replace them by R spheres S^3 glued by connected sum. Before the gluing we need to extract from each S^3 one tetrahedron:

$$N_1^{'(\ell)}\left(S^3\right) = N_1^{(\ell)}\left(S^3\right) - 1. \tag{19}$$

Here -1 represents the elimination of one tetrahedron from the triangulation of S^3. when we apply the connected sum. The other $V_{0h}^{(\ell)} - 2H_0^{(\ell)} - R$ remain unchanged.

The discrete volume (number of tetrahedra obtained by triangulation) of $M_{1h}^{(\ell)}(R)$ is

$$N_{1h}^{(\ell)}(R) = RN_1^{'(\ell)}\left(S^3\right) + \left(N_{0h}^{(\ell)} - 2N_0^{(\ell)}(H) - R\right) + N_{1h}^{(\ell)}(\Sigma) \tag{20}$$

where $N_{1h}^{(\ell)}(R)$ is the number of tetrahedra of the triangulation of level one of the sub manifold $\Sigma_1^{(\ell)}$ without the three solid torus extracted via splicing operation. Then calculating the number of tetrahedra we have:

$$N_{1h}^{(\ell)}(\Sigma) = 3 \times 6a_{11}^{(\ell)}a_{21}^{(\ell)}a_{31}^{(\ell)} + 2 \times 6a_{11}^{(\ell)}a_{21}^{(\ell)}a_{31}^{(\ell)} = 30a_{11}^{(\ell)}a_{21}^{(\ell)}a_{31}^{(\ell)}. \tag{21}$$

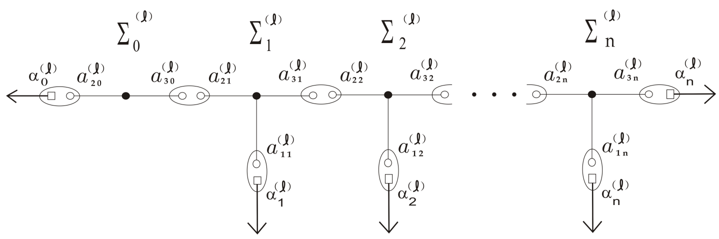
We highlight that we have removed 3 singular orbits which were replaced by hyperbolic cusp spaces, thus we only have regular orbits and points to infinity.

We characterize the assembly $E_{1h}^{(\ell)}$ by the *discrete in average volume* (*da-volume*). Namely, we suppose that the probability of gluing one sphere S^3 or one triangulated cusp hyperbolic space and the probability of leaving unchanged a tetrahedron is $1/2$ because there are only two options, that is, go to the next level or not to go. Under the assumption that we are building a model in which we contemplate it if something changes of state or not, we apply the binomial distribution in order to obtain the *da-volume* of the new manifold:

$$V_{1h}^{(\ell)} = \frac{N_1^{'(\ell)}\left(S^3\right)}{2}\left[N_{0h}^{(\ell)} - 2N_0^{(\ell)}(H)\right] + N_0^{(\ell)}(H) + 2N_1^{(\ell)}(H) + 3 \times N_{1(reg)}^{(\ell)} \tag{22}$$

where $N_{n(reg)}^{(\ell)} = 6a_{1n}^{(\ell)}a_{2n}^{(\ell)}a_{3n}^{(\ell)}$.

A generalized schematic generalization of the *da-volume* process can be done using the following splice diagram:



Note that we have removed singular orbits which have been replaced by hyperbolic cusp spaces. Thus, let's take the average due to changes of states:

$$\sum N_k^{(\ell)}(H); \tag{23}$$

with k indicating the level where is the hyperbolic cusp space. Therefore:

$$V_{nh}^{(\ell)} = \frac{N'^{(\ell)}_{n}\left(S^3\right)}{2}\left[N_{(n-1)h}^{(\ell)} - N_{(n-1)}^{(\ell)}(H) - \sum_{k=0}^{n-1} N_k^{(\ell)}(H)\right] + N_n^{(\ell)}(H) + \sum_{k=0}^{n} N_k^{(\ell)}(H) + 3 \times N_{n(reg)}^{(\ell)}. \quad (24)$$

From the steps defined up to Equation 24, it is important to remark that all singular orbits were removed and replaced either by another SF-Sphere or a cusp hyperbolic space, fulfilling our goal of avoiding singular orbits. Hereafter we will follow the theory presented in Efremov [7], [8] and Efremov & Mitskievich [10]. For gauge theory the reader can revise [24]. Observe in Table 1 that the *da-volumes* grow very fast as n changes from level 0 to 4. We associate this phenomenon with inflation processes popular in gauge theories and modern cosmological models (revised in [24]) .

Table 1: The *da-volumes* in the case of "cosmological" interactions of the $(n, 4)$-Universes

n	$V_{nh}^{(4)} = V_{nh}^{(COSM)}$	$n \in [0, 4]$
0	7.18×10^{7}	
1	1.69×10^{31}	
2	3.70×10^{83}	
3	3.39×10^{171}	
4	2.71×10^{299}	

In contrast to gauge theories where the cause of the inflation process is the change (destruction) of gauge symmetries, in our model it appears due to topology changes. To explain this we note that the steady-state or state of maximum probability of $(n-1, \ell)$-Universe has a *da-volume* characterized by the assembly $E_{n-1\,h}^{(\ell)}$.

4.1. Computing Volumes

Algorithm 3: Volumes $(n_{max}; l_{max}; \sum)$

comment: l_{max} derivation maximun order **for** $l \leftarrow 1$ **to** $l_{max}+1$

do

- **comment:** First level
- $a_{i1} \leftarrow \Sigma[1,1,l], a_{i2} \leftarrow \Sigma[1,2,l], a_{i3} \leftarrow \Sigma[1,3,l]$
- $H[1,l] \leftarrow 6\, a_{i1}\, a_{i2}\, a_{i3}$
- $S_1 \leftarrow 6\, a_{i1}\, a_{i2}\, a_{i3}$
- $Volume(1,l) \leftarrow 2 * H[1,l] + 2\, S_1$
- **comment:** Second level
- $N_{in}[1] \leftarrow 2\,(a_{i1}\, a_{i2} + a_{i2}\, a_{i3} + a_{i1}\, a_{i3}) - 1$
- $a_{i1} \leftarrow \Sigma[3,1,l], a_{i2} \leftarrow \Sigma[3,2,l], a_{i3} \leftarrow \Sigma[3,3,l]$
- $N_{in}[2] \leftarrow 2\,(a_{i1}\, a_{i2} + a_{i2}\, a_{i3} + a_{i1}\, a_{i3}) - 1$
- $N_{2r} \leftarrow 6\,(a_{i1}\, a_{i2}\, a_{i3})$
- $H[2,l] \leftarrow 6\, a_{i1}\, a_{i2}\, a_{i3}$
- $Volume(1,l) \leftarrow N_{in}[2]/2\,(Volume[1,l] - 2\, H[1,l])$
  $+H[1,l] + 2\, H[2,l] + 3\, N_{2r}$
- **comment:** Other levels
- **for** $i \leftarrow 3$ **to** $l_{max}+1$

  do
  - $a_{i1} \leftarrow \Sigma[2\,i-1,1,l]$
  - $a_{i2} \leftarrow \Sigma[2\,i-1,2,l]$
  - $a_{i3} \leftarrow \Sigma[2\,i-1,3,l]$
  - $N_{in}[i] \leftarrow 2\,(a_{i1}\, a_{i2} + a_{i2}\, a_{i3} + a_{i1}\, a_{i3}) - 1$
  - $N_{ir} \leftarrow 6\,(a_{i1}\, a_{i2}\, a_{i3})$
  - $SumH_i \leftarrow H[1,l] + H[2,l] + \cdots + H[i-1,l]$
  - $SumH_{ip} \leftarrow SumH_i + H[i,l]$
  - $Volume(i,l) \leftarrow N_{in}[i]/2\,(Volume[i-1,l]$
    $-H[i-1,l] - SumH_i)$
    $+H[i,l] + SumH_{ip} + 3N_{ir}$

We assume that the $(n-1,\ell)$-Universe is "close" to the equilibrium state and therefore has a volume approximately equal to $V^{(\ell)}_{n-1h}$.

For any spatial section which have a SF-Homologic Sphere with its respective hyperbolic cusp space (with the same section level), we define:

$$M_i := \Sigma_i \backslash_{k=1}^{3} TS_k \cup H_k \tag{25}$$

for $i = 1, 2, ..., n$ and TS_k is the corresponding tubular neighbourhood.

Now, we suppose in the equilibrium state a topology change occurs in its spatial section. For 25, we have:

$$M_0 - M_1 - \cdots - M_{n-1} \longrightarrow M_0 - M_1 - \cdots - M_{n-1} - M_n. \tag{26}$$

The changing relation given above induces a triangulation refinement from level n to level $n-1$. In terms of our model [11] of T_0-discrete universe, it means the change of the discrete volume. If we assume that the (n,ℓ)-Universe obtained tends to equilibrium (or maximum probability), then its discrete volume tends to *da-volume* $V^{(\ell)}_{nh}$, which characterizes the balance state of assembly $E^{(\ell)}_{nh}$. Note that as the volume $V^{(\ell)}_{nh} \gg V^{(\ell)}_{n-1h}$ for $n \in [0,4]$ grows (Table 1), the assemblies get proportionally smaller (Table 2).

Table 2: The *da-volume*s of the assemblies $E^{(\ell)}_{nh}$ $n \in [0,4]$.

$n\backslash l$	0	1	2	3	4
0	0.15	0.09	0.03	0.05×10^{-1}	0.01×10^{-2}
1	0.02	0.04×10^{-1}	6.40×10^{-05}	1.11×10^{-08}	3.01×10^{-16}
2	0.03	4.45×10^{-05}	3.60×10^{-09}	1.38×10^{-17}	1.96×10^{-34}
3	3.72×10^{-05}	1.36×10^{-08}	5.78×10^{-17}	2.46×10^{-34}	4.19×10^{-69}
4	3.58×10^{-08}	2.52×10^{-14}	1.07×10^{-28}	4.82×10^{-60}	9.28×10^{-123}

We note that the common inflation (in cosmological models based on gauge theories) is associated to the last change of the *da-volumes*, i.e. $\simeq 10^{299}$. The order of $V^{(4)}_{4h} \backslash V^{(4)}_{3h} \simeq 10^{128}$ agrees with the predictions of standard models for inflation [25] , also with experimental data concerning to the structure of the real universe.

It is known that in models of inflation the cosmological constant Λ (cosmological vacuum energy density [25]) is connected to the volume of constructed universe in the following way:

$$\Lambda \sim 1 \backslash \sqrt{V^{(COSM)}}. \tag{27}$$

The dimensionless cosmological constant of level n is defined by the average of the volume for the discrete manifold $E^{(4)}_{nh} = E^{(COSM)}_{nh}$ given in (27). In other words, we define the dimensionless cosmological constant of level n by the next equation:

$$\alpha^{(4)}_{nh} = 1 \backslash \sqrt{V^{(4)}_{nh}}. \tag{28}$$

The obtained values for the topological changes of this constant are shown in Table 3.

Table 3: Values for $\alpha^{(COSM)}_{nh}$ from the assemblies in the evolving Universe

$E^{(4)}_{nh}$ $n \in [0,4]$	$\alpha^{(COSM)}_{nh} = 1 \backslash \sqrt{V^{(COSM)}_{nh}}$ $n \in [0,4]$
$E^{(4)}_{0h}$	1.18×10^{-4}
$E^{(4)}_{1h}$	3.01×10^{-16}
$E^{(4)}_{2h}$	1.96×10^{-34}
$E^{(4)}_{3h}$	4.19×10^{-69}
$E^{(4)}_{4h}$	9.28×10^{-123}

If $\ell = n - \tau = 4$, $n = 4$. we have

$$\alpha^{(COSM)}_{4h} = 1 \backslash \sqrt{V^{(COSM)}_{4h}} \simeq 9.28 \times 10^{-123}. \tag{29}$$

Which agrees well with the experimental evaluation of $\Lambda\ell_{pl}^2 \sim 10^{-121}$ [26] for our universe.
We can generalize the equation (28) for all coupling constants:

$$\alpha_n^{\ell} = 1/\sqrt{V_n^{(\ell)}} \text{ for } n \in [0,4]\ ,\ \ell \in [0,4] \tag{30}$$

And assign the following names to the defined assemblies:

Strong	$E_{nh}^{(0)} = E_{nh}^{(\text{STRONG})} n \in [0,4]$
Electromagnetic	$E_{nh}^{(1)} = E_{nh}^{(\text{E\_MAG})} n \in [0,4]$
Weak	$E_{nh}^{(2)} = E_{nh}^{(\text{WEAK})} n \in [0,4]$
Gravitational	$E_{nh}^{(3)} = E_{nh}^{(\text{GRAV})} n \in [0,4]$
Cosmological	$E_{nh}^{(4)} = E_{nh}^{(\text{COSM})} n \in [0,4]$

Then, apply:

$\alpha_{nh}^{(\text{STRONG})} = 1\backslash\sqrt{V_{nh}^{(0)}}$.
$\alpha_{nh}^{(\text{E\_MAG})} = 1\backslash\sqrt{V_{nh}^{(1)}}$,
$\alpha_{nh}^{(\text{WEAK})} = 1\backslash\sqrt{V_{nh}^{(2)}}$,
$\alpha_{nh}^{(\text{GRAV})} = 1\backslash\sqrt{V_{nh}^{(3)}}$,
$\alpha_{nh}^{(\text{COSM})} = 1\backslash\sqrt{V_{nh}^{(4)}}$,

We show the predicted values for the coupling constants for $\tau = 0$ (here τ is the redshift-like domain) in Table 4, along with recent experimental values for these constants. The hierarchy coincides fairly.

Table 4: The correspondence of the dimensionless coupling constants vs. experimental values.

k	α *coupling*	*Interaction*	α *experimental*	Reference
0	0.15	strong	0.11	[27]
1	3.91×10^{-3}	electromagnetic	$7,29 \times 10^{-3}$	[28]
2	3.60×10^{-9}	weak	1.20×10^{-5}	[28]
3	1.96×10^{-34}	gravitational	$1,75 \times 10^{-45}$	[28]
4	9.28×10^{-123}	cosmological	1.70×10^{-121}	[26]

5. Concluding Remarks

In our construction we removed the tubular neighbourhoods of singular orbits and replaced them by hyperbolic spaces, thus achieving two things: First, we could avoid singularities from space, getting closer to a physical representation. Namely this represents a new approach for a Cosmological Universe Model without Friedmann type singularities. Usually it is hard to have a physical representation when in theory we have a singularity. Instead, we can imagine singular points as points that go to infinite. Second, we established an approach for calculating coupling constants in terms of hierarchy, all based in Knesser, Milnor, Jaco, Shalen, Johanson and Thurston's theory.

We constructed a manifold characterized by an infinite space but with finite volume (following [29]). This volume was defined by counting the number of tetrahedra in the manifold. However, we may note that the calculation of the exact volume of ideal tetrahedron is still a subject of mathematical research. Thus, although we could use this information in the specific case presented here, we are not able to calculate the total volume of the Universe. When a specific formula for the volume of an ideal tetrahedron is found, this goal can be achieved.

With the obtained discrete volumes we calculated the coupling constants at $\tau = 0$. Our calculations resulted in a $\alpha_{nh}^{(COSM)} = \alpha_\Lambda = 9.28 \times 10^{-123}$ for the evolving Universe which in our model, we asociated with the inflation, and a value of 1.70×10^{-121} Plank units for Λ, in rough accordance with the observed values, according to [30] .

Since with the present work we were able to obtain a successful representation of cosmological properties using a geometric-topological approach, the next step consists in constructing a completely hyperbolic manifold as a possible representation of our Universe. The aim of this endeavour is to possibly explain important cosmological puzzles, such as the Big-Bang, the inflation and the dark energy, as direct consequences of the geometry-topology of the Universe.

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